Commitee decisions: optimality and equilibrium

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Abstract. We consider a committee facing a binary decision under uncertainty. Each member holds some private information. Members may have different preferences and initial beliefs, but they all agree which decision should be taken in each of the two states of the world. We characterize the optimal anonymous and deterministic voting rule and provide a homogeneity assumption on preferences and beliefs under which sincere voting is a Nash equilibrium for this rule. We also provide a necessary and sufficient condition for sincere voting to be an equilibrium under any deterministic majoritarian voting rule. We show that a class of slightly randomized majoritarian voting rules make sincere voting a strict and unique equilibrium. A slight deontological preference for sincere voting, or *ex post* revelation of individual votes — "transparency" — combined with a concern for esteem, has the same effect.

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1. Introduction

We here analyze a generalized "Condorcet jury problem," that is, a situation in which a decision has to be taken collectively by a committee or jury. There are only two states of the world. In the case of a jury, the defendant is either guilty or innocent. However, the state is unknown at the time of the decision and individual jury members have private information about the true state. Condorcet's (1785) classical theorem essentially establishes that, under conditionally independent private information, aggregation by way of voting under majority rule is asymptotically efficient in the sense that the probability for a mistaken jury decision — convicting an innocent defendant or acquitting a guilty — tends to zero as the number of jury members goes to infinity.

The modern strategic analysis of jury voting, pioneered by Austen-Smith and Banks (1996), has pointed out a major weakness of the classical result. An unstated hypothesis underlying Condorcet's analysis is that jury members vote informatively, that is, base their votes solely on their own private information, without regard to other jury members' potential information and votes. This hypothesis may seem innocuous, since all jury members share the same goal. However, a careful analysis shows that this is not always the case. More exactly, Austen-Smith and Banks noticed that, if the number of jury members is large, informative voting is in general not a Nash equilibrium of the Bayesian game that corresponds to Condorcet's setting.

This negative result emanates from the following observation: An individual vote makes a difference only if it is pivotal. Hence, as a voter under majority rule I should reason as if I knew that the other votes were in a tie.[1] But if there are many voters, the (hypothetical) fact that all the others are tied is very informative, perhaps "drenching" my own private information. Under such weak "evidence," perhaps I should not vote according to my private information. If the committee, jury or electorate is large enough, this argument against informative voting becomes overwhelming. Consequently, informative voting is then not a Nash equilibrium, and Condorcet's judgement-aggregation argument fails.

We here generalize somewhat the setting and address some more issues, allowing for preference and belief heterogeneity, so that committee members may differ in the von Neumann utilities they attach to the different combinations of decisions and

[1] The notion of a pivotal event for a player is not restricted to voting games; Al-Najjar and Smorodinsky (2000) defined, in a rather general setting, the *influence* of a player in a mechanism as the maximum difference this player's action can make in the expected value of a collective result. They show that, a precise sense, the mechanisms that maximise the number of influential players are derived from majority rule.

states of the world, as well as in the subjective probabilities they attach to these states. In particular, committee members need not agree precisely about the relative "costs" associated with the two types of mistake that can be made — taking decision 1 in state 0 or decision 0 in state 1. We assume, however, that all committee members are equally "competent" in the sense that their private information is equally precise.

In this somewhat generalized setting, we first characterize optimal anonymous and deterministic voting rules in a committee of arbitrary finite size. We show that if the committee is sufficiently homogeneous in terms of preferences and beliefs, then informative voting is an equilibrium under the optimal voting rule. As a corollary we obtain that, contrary to what one might expect, the optimal voting rule for large committees or electorates does not depend on aggregate preferences and beliefs, only on the precision of the private information. Secondly, we re-examine Austen-Smith's and Banks' (1996) result, and, thirdly, following Coughlan (2000), we analyze the effect of having a straw vote before the decisive vote and establish that if the committee is sufficiently homogeneous in terms of preferences and beliefs, informative voting is consistent with equilibrium. More exactly, if each committee member believes that all others vote informatively in the straw vote, then no committee member has an incentive to deviate from informative voting, and once the total number of straw votes for the two alternatives have been made public, all committee members will vote in the same way in the decisive vote. This does not hold, however, if committee members' preferences or beliefs are sufficiently heterogeneous, as we demonstrate by way of a numerical example; then committee members with extreme preferences or initial beliefs may have an incentive not to reveal their private information in the straw vote. Moreover, even when informative voting is an equilibrium in this setting, there is still a plethora of uninformative equilibria, so straw vote is not a universal remedy.

Fourthly, we introduce a one-stage voting procedure, a slightly randomized majority rule, and show that informative voting then is a strict Nash equilibrium and that this is the unique equilibrium. We also show that this slightly randomized voting rule is asymptotically efficient: in the limit as the committee size goes to infinity, the probability of a mistaken collective decision tends to zero.

Fifthly, and finally, we reconsider majority rule in the basic setting, but now applied to committee members who slightly dislike voting against their personal beliefs and/or who care about their esteem, which depends on *ex post* observation (by the general public or the committee members) of the true state (at the time of the decision) and their individual vote. Such a voter is rational but has slightly different preferences from what is usually assumed in voting models; a deontological preference against insincere voting. Irrespective of how weak this preference for sincerity is, there exists a critical committee size such that sincere voting is a strict Nash equilibrium whenever the committee is large enough. This is the unique equilibrium and its outcome is asymptotically efficient; the probability of a mistaken collective decision goes to zero as the committee size tends to infinity. Hence, in so far that real-life committee members either have a deontological preference for sincerity or the voting procedure is ex post transparent, collective decisions are likely to aggregate judgements more efficiently than suggested by current theory, even when voters act strategically.

Before embarking upon our analysis, let us comment on related research. The two seminal papers on incentives for informative voting are Austen-Smith and Banks (1996), mentioned above, and Feddersen and Pesendorfer (1996). In the latter paper, the so-called swing voter's curse is analyzed. It refers to the following phenomenon. If voters, among whom there are partisans for each alternative as well as non-partisans, are allowed to abstain from voting, then poorly informed non-partisans may use the following mixed strategy. They probabilistically balance their votes in such a way that they collectively compensate for the presence of partisan voters (who support a given candidate in any case) and leave room for the better informed non-partisan voters. This mixed strategy of poorly informed non-partisan voters involves abstention with positive probability. By contrast, we do not allow our voters to abstain — this is one of the two senses in which our model is about committees, as opposed to electorates.

Subsequent theoretical research on jury behavior mainly concern the relative merits of different voting rules, see Feddersen and Pesendorfer (1998) and the role of straw-votes or debates before voting. Coughlan (2000) was mentioned above, and Austen-Smith and Feddersen (2005) is another contribution along similar lines. In these models, where voters are identical, the picture is very different with and without debate or straw vote. Moreover, if voters with identical preferences and priors share their private information, then decisions are unanimous in the decisive vote, and all majoritarian voting rules (including unanimity) are equivalent, see also Gerardi and Yariv (2007). These results do not hold if voters are not identical.

The second sense in which our model concerns committees and not electorates is that we assume that the number of voters is fixed and known. By contrast, in real-life elections, this number is usually unknown at the time of voting. See Myerson (1998), Feddersen and Sandroni (2002) and Krishna and Morgan (2007) for excellent studies of abstention and random electorates. We here focus exclusively on the binary decision problems, but there has been a recent renewal of interest and new results have been obtained for more general collective decision problems under uncertainty, see McLennan (2007).

The rest of the paper is organized as follows. In section 2 we set the stage for the subsequent analysis by way of spelling out the base-line model. Section 3 analyzes optimality of anonymous voting rules. Section 4 is devoted to equilibrium considerations and establishes a slight generalization of Theorem 1 in Austen-Smith and Banks (1996). Section 5 examines a two-stage voting procedure where the first stage is a straw vote. Section 6 develops a slightly randomized majority rule and we show that informative voting is the unique Nash equilibrium under this rule, and that this equilibrium outcome is asymptotically efficient. Essentially the same conclusions are reached in section 7 for voters with a slight preference for sincere voting under one-stage majority rule. Section 8 concludes. Technical proofs have been relegated to an appendix at the end of the paper.

2. THE FRAMEWORK

There are n committee members, where n is a positive integer. The committee has to make a binary decision, $x \in \{0,1\} = X$. All committee members agree what is the right decision in each state of nature. However, they do not know the state of nature $\omega \in \{0,1\} = \Omega$. Each committee member i has a prior belief about the actual state of nature. These priors may be, but need not be, the same for all committee members.[2] Moreover, such priors may be based in part on a common signal, received by all committee members, as, for example, in the proceedings during a trial, during a committee hearing, or a public debate before an election. We here analyze the decision problem faced by the committee (jury or electorate) after any common signal has been received and we refer to the committee members' beliefs at that stage as their priors.

Formally, let μ_i be the prior probability that committee member i assigns to state $\omega = 1$ (and thus $1 - \mu_i$ to state $\omega = 0$) and assume that no state is *a priori* excluded; $0 < \mu_i < 1$ for all committee members i. In addition, each committee member i receives a private "signal" $s_i \in \{0,1\}$, a random variable that is positively correlated with the true state of nature ω:

$$\begin{cases} \Pr[s_i = 0 \mid \omega = 0] = q_0 \\ \Pr[s_i = 1 \mid \omega = 1] = q_1 \end{cases}$$

[2] See Dixit and Weibull (2007) for an analysis of how judgmental polarization can arise among voters with identical preferences but distinct priors.

for $q_0, q_1 > 1/2$. Hence, all committee members have the same "competence" — or receive signals of the same "quality" — in the sense that, in a given state, they all have the same conditional probability of receiving a correct signal. Signals received by different committee members are, however, conditionally independent, given the state of nature.

All committee members agree that the right decision in state ω is $x = \omega$. In the jury example, they all wish the decision to be conviction if the defendant is guilty and acquittal if the defendant is innocent. In the investment decision, they wish the investment to be made if and only it is profitable. However, in all other respects they may differ in the von Neumann-Morgenstern utilities that they assign to the four possible decision-state pairs. Without loss of generality, these utilities are given for each committee member i by the following table:

	$\omega = 0$	$\omega = 1$
$x = 0$	u_{i0}	$u_{i1} - \alpha_i$
$x = 1$	$u_{i0} - \beta_i$	u_{i1}

In this notation, the assumption that they all agree about what is the right decision in each state simply means that both α_i and β_i are positive, an assumption that we will maintain throughout this study. For each committee member i, these two parameters are the disutilities that the committee member attaches to the two types of mistake, namely, of taking the wrong decision in state $\omega = 1$ (say, acquitting a guilty defendant or passing a profitable investment opportunity) and of taking the wrong decision in state $\omega = 0$ (say, convicting an innocent defendant or making an unprofitable investment). We will sometimes refer to the first mistake (decision $x = 0$ in state $\omega = 1$) as a mistake of type I (accepting the false null hypothesis that the state is 0) and the second mistake (decision $x = 1$ in state $\omega = 0$) as a mistake of type II (rejecting the true null hypothesis that the state is 0).

For many purposes, the relevant data about each committee member i can be summarized in one number, namely

$$\gamma_i = \frac{\alpha_i \mu_i}{\beta_i (1 - \mu_i)} \tag{1}$$

where $\gamma_i > 0$ follows from our assumptions. Note that $\gamma_i = 1$ if and only if both types of mistake carry the same *ex ante* expected utility *loss*. Before receiving one's signal, the probability that committee member i attaches to state 1 is μ_i and the "cost" of a mistake then (that is, a mistake of type I) is α_i. Hence, the *ex-ante* expected utility loss to committee member i of a mistake of type I is $\mu_i \alpha_i$. The probability attached to state 1 is $1-\mu_i$ and the "cost" of a mistake then (that is, of type II) is β_i. Acquitting an guilty defendant or passing a good investment opportunity may often be a lesser mistake than convicting an innocent defendant or making a bad investment. In such cases $\alpha_i < \beta_i$ and hence $\gamma_i < 1$ if the prior is uniform $\mu_i = 1/2$.[3]

In the base-line setting, each committee member i casts a vote $v_i \in \{0,1\}$, a vote which may, but need not, be guided by i's private signal, and the collective decision x is determined by way of some pre-specified rule f that maps each vote profile $v = (v_1, ..., v_n)$ to a probability $f(v) \in [0,1]$ that the decision will be $x = 1$. The probability for decision $x = 0$ thus is assigned probability $1 - f(v)$. Formally, a *voting rule* is a function $f : \{0,1\}^n \to [0,1]$.

A *voting strategy* for committee member i in the base-line setting is a function $\sigma_i : \{0,1\} \to [0,1]$ that maps i's signal s_i to a probability $\sigma_i(s_i)$ for a vote v_i on alternative 1: $\Pr[v_i = 1 \mid s_i] = \sigma_i(s_i)$.[4] In others words, a voting strategy prescribes with what probability the committee member will vote for decision alternative 1 (and vote on alternative 0 with the residual probability). By a pure voting strategy we mean a strategy σ_i such that $\sigma_i(s_i) \in \{0,1\}$ for both signals s_i. In this case, $v_i = \sigma_i(s_i)$.

In the voting literature, the pure strategy to always vote according to one's signal, $\sigma_i(s_i) \equiv s_i$, is usually called *informative* voting, while voting for the alternative that maximizes the voter's expected utility, conditional on her own signal, and only on that piece of information, is called *sincere* voting.

Each committee member i is thus characterized by the parameter triplet $\theta_i = (\alpha_i, \beta_i, \mu_i)$, where all three parameters are positive and $\mu_i < 1$. When studying asymptotic properties of arbitrarily large committees, we will assume that all parameter triplets belong to some compact set in the interior of the parameter space:

$$\theta_i \in \Theta = [\alpha_{\min}, \alpha_{\max}] \times [\beta_{\min}, \beta_{\max}] \times [\mu_{\min}, \mu_{\max}] \tag{2}$$

where all bounds are positive and $\mu_{\max} < 1$. We will refer to this condition as the preference boundedness condition. This condition is met by the two standard methods to theoretically generate ever larger economies: (i) by replicating a given finite set of

[3]In environmental management, type II errors can be more costly than type I errors for environmental management. This is because the commitment of time, energy and people to fighting a false alarm (a type I error) may be short term (until the mistake is discovered), while the cost of not doing something when in fact it should be done (a type II error) will have both short- and long-term costs (e.g. ensuing environmental degradation).

[4]We will later analyze behavioral voting strategies under two-stage voting rules.

economic agents, or (ii) by independent sampling from a fixed probability distribution over the type space, here Θ.

2.1. Condorcet's jury theorem. Condorcet's Jury Theorem, stated in terms of the present model, asserts that if all committee members vote informatively, then the probability of a mistaken collective decision under majority rule tends to zero as the committee size tends to infinity. The result hinges on the conditional independence of the signals and the assumption that they are positively correlated with the true state. The result does not depend on the committee members' preferences and beliefs, since their voting behavior is assumed and that is all that matters:

Theorem 1 [Condorcet]**.** *Suppose that all committee members vote informatively under majority rule. Let $X_n(\omega) \in \{0,1\}$ be the collective decision when there are n committee members and the true state is ω. Then*

$$\lim_{n\to\infty} \Pr[X_n(\omega) \neq \omega \mid \omega] = 0 \quad \forall \omega \in \Omega$$

For the sake of completeness, we provide a modern proof in the appendix.

2.2. Signal informativeness. The Condorcet theorem presumes that all committee members vote informatively. Clearly, this is not always a reasonable assumption, not even for $n = 1$, a single decision-maker. To clarify this aspect, suppose, that one committee member has been selected to make the decision single-handedly, based only on his or her private signal. If the signal is noisy and her prior and valuation of mistake costs favor one alternative over the other, the right decision may well be to disregard the signal. More exactly, an application of Bayes' rule gives the following posterior probability for state 0 after signal 0 has been received (recall that the prior is $1 - \mu_i$):

$$\Pr[\omega = 0 \mid s_i = 0] = \frac{(1-\mu_i)\Pr[s_i = 0 \mid \omega = 0]}{\Pr[s_i = 0]} = \frac{(1-\mu_i)\, q_0}{(1-\mu_i)\, q_0 + \mu_i (1-q_1)}$$

and likewise when the signal $s_i = 1$ has been received. Consequently, the strategy to vote informatively, $v_i \equiv s_i$, is optimal if and only if $(1-\mu_i)\, q_0 \beta_i \geq \mu_i (1-q_1)\, \alpha_i$ and $\mu_i q_1 \alpha_i \geq (1-\mu_i)(1-q_0)\, \beta_i$, or, equivalently, if and only if $(1-q_0)/q_1 \leq \gamma_i \leq q_0/(1-q_1)$. We will assume that both inequalities hold strictly for all committee members,

$$\frac{1-q_0}{q_1} < \gamma_i < \frac{q_0}{1-q_1} \qquad \forall i, \tag{3}$$

a condition we will refer to as the signal informativeness condition. It follows from our assumption $q_0, q_1 > 1/2$ that the lower (upper) bound in (3) is below (above) 1.

For the sake of illustration of this condition, consider a court jury member i who *a priori* believes that both states (that the defendant is innocent or guilty, respectively) are equally likely, $\mu_i = 1/2$, and who finds convicting an innocent defendant twice as bad a mistake as acquitting a guilty defendant, $\beta_i = 2\alpha_i$. Then $\gamma_i = 1/2$ and thus the right-hand inequality in (3) is always met: the committee member should never vote against the signal "innocent." However, the left-hand inequality is violated if (and only if) $q_0 + q_1/2 < 1$, that is, if the signal is not so strongly correlated with the true state of the world (say, $q_0 = q_1 = 0.6$). It is then optimal is "guilty" ($s_i = 1$).

Remark 1. *Let f be a voting rule that randomly picks 1 vote out of the n votes cast, with a positive and exogenously fixed probability for each vote v_i to be picked, and let the collective decision x be determined by that vote: $x = v_i$. Under such a randomized voting rule, a rational committee member realizes that, independently of how others in the committee vote, his or her vote will matter only if selected. Since, moreover, the probability for this event is positive and independent of his or her own action, sincere voting is optimal under condition (3). Under such a randomized voting rule, sincere voting is not only compatible with Nash equilibrium; it is a dominant strategy.*

3. Optimal voting rules

What voting rules are optimal for the committee? We here briefly consider the optimality of deterministic one-stage voting rules, that is, voting rules $f : \{0, 1\}^n \to \{0, 1\}$ that map vote profiles $v = (v_1, ..., v_n)$ to decisions (randomized voting rules will be considered in Section 6).

As for normative criteria by which to define optimality, the following two seem most relevant: maximization of the (subjective or objective) probability for taking the right decision, or, alternatively, maximization of the sum of the committee members (subjectively or objectively) expected utility from the decision. While the first criterion does not discriminate between mistakes of type I and II, the second does. And this makes sense in many contexts. For instance, if most or all members of a jury consider it a worse error to convict an innocent than to acquit a guilty defendant, then it is desirable that the voting rule accounts for this preference asymmetry. We here focus on this latter, utilitarian criterion. We call such a function *optimal* if there exists no other such function that yields higher expected welfare. Formally, for any deterministic voting rule f, let

$$\begin{aligned} W(f) &= \sum_{i=1}^{n} u_0^i \Pr[x=\omega=0 \mid \mu_i] + \sum_{i=1}^{n} \left(u_0^i - \beta_i\right) \Pr[x=1,\omega=0 \mid \mu_i] \\ &\quad + \sum_{i=1}^{n} u_1^i \Pr[x=\omega=1 \mid \mu_i] + \sum_{i=1}^{n} \left(u_1^i - \alpha_i\right) \Pr[x=0,\omega=1 \mid \mu_i] \end{aligned}$$

where $x = f(s_1, ..., s_n)$. With some abuse of notation, $\Pr[\cdot \mid \mu_i]$ here denotes the probability for an event " $\cdot$ " under the prior μ_i. All these priors may be the same and this common prior may even be an objective probability.

Hence, we evaluate welfare when the voting rule f in question is applied directly to the signal vector, or, equivalently, when all committee members vote informatively. Optimality in this first-best sense is then defined as maximization of W over the set of deterministic voting functions. An optimal voting rule is thus an *optimal deterministic direct mechanism*, with no requirement of incentive-compatibility.[5]

Of great practical relevance is the subset of so-called k-majority rules, rules that require at least k votes out of n in order for decision 1 to be taken. Formally, let $\mathbb{N}$ be the nonnegative integers and for any $k \in \mathbb{N} \cap [0, n+1]$, let $f^k : \{0,1\}^n \to \{0,1\}$ be the *k-majority rule* defined by $f^k(v_1, ..., v_n) = 1$ iff $\sum_{i=1}^n v_i \geq k$. For n odd, simple majority rule is thus the special case when $k = (n+1)/2$. For arbitrary n, $k = 1$ and $k = n$ are the two *unanimity rules* (requiring n votes for decision 0 and 1, respectively), $k = 0$ the rule to take decision 1 irrespective of the votes and $k = n+1$ the rule to take decision 0 irrespective of the votes. It is not difficult to verify that if a k-majority rule is optimal and informativeness condition (3) holds, then $1 \leq k \leq n$ and no other deterministic voting rule can result in higher welfare.[6]

Hence, without loss of generality we may restrict the quest for optimal rules to k-majority rules and focus on $k \in \mathbb{N} \cap [1, n]$. Since the number n of committee members is finite, existence of an optimal voting rule is guaranteed. The following result provides a necessary and sufficient condition for optimality. In order to state

[5] As shown in Chwe (2007), optimality under the incentive constraints for sincere voting does not necessarily, or even typically, lead to monotonic voting rules.

[6] To see this, suppose $f : \{0,1\}^n \to \{0,1\}$ is optimal. Since all voters' signals have the same precision, there exists some symmetric function $g : \{0,1\}^n \to \{0,1\}$ such that $W(g) = W(f)$. Then $g(s_1, ..., s_n)$ is a function h of the signal sum $\sum s_i$. Since f maximizes W, so does g, and then h has to be increasing, since $q_0, q_1 > 1/2$ by assumption, so nothing can be gained by disregarding a signal. Since h is increasing, g is a k-majority rule for some $k \in \{0, 1, ..., n, n+1\}$. If the signal informativeness condition (3) holds strictly, and $\alpha_i, \beta_i > 0$, it is never optimal to disregard all signals, so then $k \in \{1, ..., n\}$.

it, let $\bar{\alpha}_n = \sum_{i=1}^{n} \mu_i \alpha_i$, $\bar{\beta}_n = \sum_{i=1}^{n} (1 - \mu_i) \beta_i$ and $\bar{\gamma}_n = \bar{\alpha}_n / \bar{\beta}_n$. Moreover, for any integers n and k, let

$$A_{k,n} = \left(\frac{1 - q_0}{q_1} \right)^k \left(\frac{q_0}{1 - q_1} \right)^{n-k}$$

and note that $A_{k,n}$ is increasing in n and decreasing in k.[7]

Theorem 2. *For any positive $n \in \mathbb{N}$ and $k \in \mathbb{N} \cap [1, n]$, k-majority rule is optimal if and only if*

$$A_{k,n} \leq \bar{\gamma}_n \leq A_{k-1,n}. \tag{4}$$

(Proof in appendix.)

Consider a sequence of committees, of ever larger size $n = 1, 2...$, all with the same signal precisions, q_0 and q_1. By our preference boundedness condition (2) there exist numbers $\gamma_{\min}, \gamma_{\max} > 0$. such that $\gamma_{\min} \leq \bar{\gamma}_n \leq \gamma_{\max}$ for all $n \in N$. In other words, the parameter sequence $(\bar{\gamma}_n)_{n \in \mathbb{N}}$ is bounded away from zero and from plus infinity, a condition that holds trivially if all committee members have the same preferences and beliefs. For each positive integer n, let $k^* (n)$ be optimal, and write $\rho^* (n)$ for the optimal vote ratio $k^* (n) / n$. It is not difficult to verify that $\rho^* (n)$ converges as the committee size n goes to infinity, and, perhaps more surprisingly, that the limit is independent of voters' preferences and priors, as long as these are bounded in the above sense. The limit of $\rho^* (n)$, as n tends to infinity, then depends only on the precision of the two signals. Moreover, the limit value of $\rho^* (n)$ never falls short of $1 - q_0$ (the probability that signal 1 is received when the state is 0) and it never exceeds q_1 (the probability that signal 1 is received when the state is 1). In particular, the limit value is 1/2 if $q_0 = q_1$. In other words: if the two signals are equally precise, then simple majority rule is optimal for large committees, independent of preferences and beliefs. Even if one type of mistake (say, convicting an innocent defendant) is deemed much worse than the other type (acquitting a guilty defendant), majority rule is still the optimal voting rule if the committee (jury or electorate) is very large:

Corollary 1. *For all $q_0, q_1 \in \left(\frac{1}{2}, 1 \right]$ and any sequence $(\bar{\gamma}_n)_{n \in \mathbb{N}}$ that is bounded away from zero and plus infinity:*

$$1 - q_0 \leq \lim_{n \to \infty} \rho^* (n) = \frac{\ln \left(\frac{q_0}{1 - q_1} \right)}{\ln \left(\frac{q_0}{1 - q_1} \right) + \ln \left(\frac{q_1}{1 - q_0} \right)} \leq q_1 \tag{5}$$

[7] Also note that $A_n \leq (1 - q_0) / q_1$ and $A_0 \geq q_0 / (1 - q_1)$. Hence the optimality condition (4) holds for some $k \in \{1, ..., n\}$, under informativeness condition (3).

The reason for the apparently counter-intuitive result that the optimal voting rule is independent of preferences can be explained as follows. Suppose that, for a given committee size n, a certain k-majority rule f^k is optimal: $k^*(n) = k$. Then we necessarily have $W(f^k) \geq W(f^{k+1})$, that is, it should not be welfare improving to instead use $(k+1)$-majority rule. Let N_1 be the (random) number of signals 1 among the n signals received. Comparing voting rules f^k and f^{k+1}, it is clear that the first is (weakly) better than the second if and only if it is better when $N_1 = k$, this being the only event in which the two voting rules differ. Moreover, if $N_1 = k$, then f^k is (weakly) better than f^{k+1} if an only if erring under voting rule f^{k+1} (taking decision 0 when the state is 1) is no smaller than the social cost of erring under f^k (taking decision 1 when the state is 0), which, formally, amounts to the inequality

$$\bar{\alpha}_n \Pr[N_1 = k \mid \omega = 1] \geq \bar{\beta}_n \Pr[N_1 = k \mid \omega = 0].$$

For very large n and proportionately large k, $k = k^*(n) \approx \rho^*(n)\, n$, the ratio between the probabilities either tends to zero or to plus infinity (since $q_0, q_1 > 1/2$). However, the ratio $\bar{\gamma}_n = \bar{\alpha}_n/\bar{\beta}_n$ is, by hypothesis bounded away from zero and plus infinity (uniformly in n). Hence, asymptotically, it does not matter exactly what values the parameters $\bar{\gamma}_n$ have, as long as they all lie in a bounded interval of positive numbers. Consequently, the limit ratio $\rho^*(n)$ does not depend on the committee members' preferences or beliefs. (The same reasoning can be applied to the necessary inequality $W(f^k) \leq W(f^{k+1})$, and a formal proof of the corollary is given in the appendix.)

While the optimal voting rule for very large committees does not depend on preferences or beliefs, this is (of course) not true for small and medium-sized committees. This is shown in the following numerical example.

Example 1. *See diagram below, drawn for $\bar{\gamma} = 1$, $q_0 = 0.8$ and $q_1 = 0.7$. On the horizontal axis is n, the number of committee members, and on the vertical axis k, the number of votes 1 for taking the collective decision 1. The two solid straight lines are the upper and lower bounds on k for optimality. We see that simple majority is optimal in the range of the diagram: $k^*(2) = 1$, $k^*(3) = k^*(4) = 2$, $k^*(5) = k^*(6) = 3$ and $k^*(7) = 4$. If $\bar{\gamma}$ is increased, then one would expect the required number of votes for decision 1 to decrease, for n fixed. The dashed straight lines are the upper and lower bounds on k for optimality when $\bar{\gamma} = 1.35$. We see that then $k^*(n)$ is indeed smaller for certain n. For instance, for $n = 7$ it now takes only 3 votes, instead of 4, for decision 1. Recall, however, that the two sequences of optimal k-majority rules,* *the one for* $\bar{\gamma} = 1$ *and the one for* $\bar{\gamma} = 1.35$ *have the same limit for* $k^*(n)/n$. *In force of the corollary, we know that this limit is approximately* 0.44.

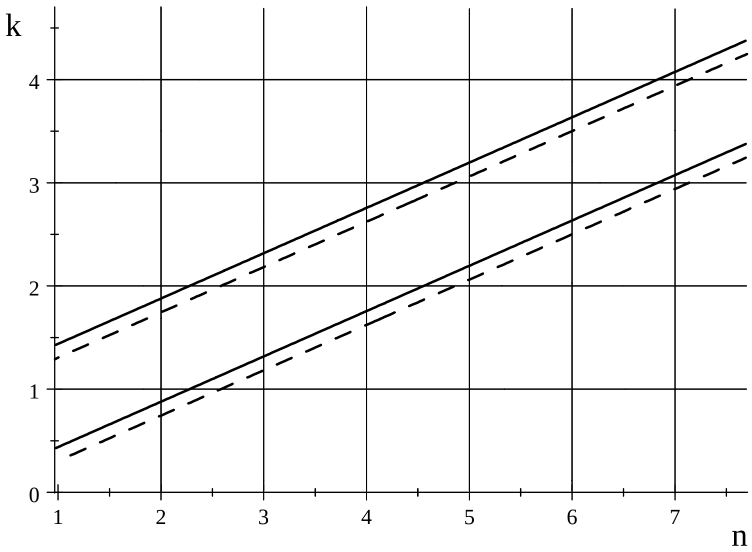

Figure 1: The lower and upper bounds on $k^*(n)$.

4. Equilibrium

Suppose that the collective decision is to be taken according to k-majority rule among n committee members, for some positive integer $k \leq n$, and as described above. This k may be, but need not be optimal. Suppose that each committee member votes so as to maximize his or her own expected utility as defined above. Is sincere voting then a Nash equilibrium? Does this depend on whether or not the voting rule is optimal? We will throughout this section assume that the signal informativeness condition (3) is met. As a consequence, sincere voting is identical with informative voting, and we will use these two attributes interchangeably. Given her signal, each voter i then casts a vote $v_i \in \{0, 1\}$, simultaneously with the other voters, and the decision $x = 1$ results if at least k voters cast the vote 1, while the decision $x = 0$ results in the opposite case.

In Nash equilibrium, each voter maximizes his or her expected utility, given his or her private signal, and given all other voters' strategies. Clearly, there is a plethora of (pure and mixed) uninformative Nash equilibria of this voting game whenever $n \geq 3$. For example, to always vote 0 (or 1), independently of one's private signal, constitutes a Nash equilibrium. For if others vote according to such a strategy, then my vote will never be pivotal and hence I can just as well use the same uninformative voting strategy as the others. Under what conditions, if any, will sincere voting constitute an equilibrium? In order to answer this question, suppose that voter i expects all others to vote sincerely and hence informatively. Is it then in i's interest to also vote sincerely?

Denote by $\mathcal{T}$ the event of a tie among the others, that is, that exactly $k-1$ of the other voters receive the signal 1 and thus $n-k$ receive the signal 0. Suppose, for instance, that i received the signal $s_i = 0$. Should i then vote on alternative 0? The probability for the joint event that $s_i = 0$ and that there is a tie among the others, conditional on the state $\omega = 0$, is

$$\Pr\left[\mathcal{T} \ \wedge \ s_i = 0 \mid \omega = 0\right] = \binom{n-1}{k-1} q_0^{n-k} \left(1-q_0\right)^{k-1}$$

Likewise, conditional on the state $\omega = 1$, we have

$$\Pr\left[\mathcal{T} \ \wedge \ s_i = 0 \mid \omega = 1\right] = \binom{n-1}{k-1} q_1^{k-1} \left(1-q_1\right)^{n-k}$$

Therefore, according to i's prior, the probability for the joint event that i receives the signal 0 and there is a tie among the others is

$$\Pr\left[\mathcal{T} \ \wedge \ s_i = 0\right] = \binom{n-1}{k-1} \left[\left(1-\mu_i\right) q_0^{n-k} \left(1-q_0\right)^{k-1} + \mu_i q_1^{k-1} \left(1-q_1\right)^{n-k}\right]$$

Since, according to i's prior, the probability of receiving the signal 0 is $\Pr\left[s_i = 0\right] = \left(1-\mu_i\right) q_0 + \mu_i \left(1-q_1\right)$, committee member i attaches the following conditional probability of a tie among the others, conditional upon $s_i = 0$:

$$p_0(m) = \Pr\left[\mathcal{T} \mid s_i = 0\right] = \binom{n-1}{k-1} \frac{\left(1-\mu_i\right) q_0^{n-k} \left(1-q_0\right)^{k-1} + \mu_i q_1^{k-1} \left(1-q_1\right)^{n-k}}{\left(1-\mu_i\right) q_0 + \mu_i \left(1-q_1\right)}$$

We are now in position to compute the difference in expected utility for voter i between casting the sincere vote $v_i = 0$ instead of the insincere vote $v_i = 1$, conditional upon the signal $s_i = 0$:

$$\Delta u_i = \mathbb{E}\left[u_i \mid s_i = v_i = 0\right] - \mathbb{E}\left[u_i \mid s_i = 0 \ \wedge \ v_i = 1\right]$$

Because i's vote affects the collective decision x only in the event $\mathcal{T}$, we have

$$\Delta u_i = p_0(m) \cdot \left(\mathbb{E}\left[u_i \mid \mathcal{T} \ \wedge \ s_i = v_i = 0\right] - \mathbb{E}\left[u_i \mid \mathcal{T} \ \wedge \ s_i = 0 \ \wedge \ v_i = 1\right]\right)$$

where

$$\begin{aligned}\mathbb{E}\left[u_i \mid \mathcal{T} \ \wedge \ s_i = v_i = 0\right] &= \kappa_i - \alpha_i \Pr\left[\omega = 1 \mid \mathcal{T} \ \wedge \ s_i = 0\right] \\ \mathbb{E}\left[u_i \mid \mathcal{T} \ \wedge \ s_i = 0 \ \wedge \ v_i = 1\right] &= \kappa_i - \beta_i \Pr\left[\omega = 0 \mid \mathcal{T} \ \wedge \ s_i = 0\right]\end{aligned}$$

and κ_i is the conditionally expected utility of taking the right decision, $x = \omega$, conditional on the event $\mathcal{T} \wedge s_i = 0$.[8] By Bayes' law (factorials cancel):

$$\begin{aligned}\Pr\left[\omega = 0 \mid \mathcal{T} \ \wedge \ s_i = 0\right] &= \frac{(1-\mu_i)\Pr\left[\mathcal{T} \ \wedge \ s_i = 0 \mid \omega = 0\right]}{\Pr\left[\mathcal{T} \ \wedge \ s_i = 0\right]} \\ &= \frac{(1-\mu_i)\, q_0^{n-k+1}\,(1-q_0)^{k-1}}{(1-\mu_i)\, q_0^{n-k1}\,(1-q_0)^{k-1} + \mu_i q_1^{k-1}\,(1-q_1)^{n-k+1}}.\end{aligned}$$

Hence:

$$\Delta u_i = \binom{n-1}{k-1} \frac{(1-\mu_i)\, q_0^{n-k+1}\,(1-q_0)^{k-1}\,\beta_i - \mu_i q_1^{k-1}\,(1-q_1)^{n-k+1}\,\alpha_i}{(1-\mu_i)\, q_0 + \mu_i\,(1-q_1)}. \tag{6}$$

The condition for Δu_i to be nonnegative, that is, for i to rationally want to vote according to her signal $s_i = 0$, is thus

$$(1-\mu_i)\, q_0^{n-k+1}\,(1-q_0)^{k-1}\,\beta_i \geq \mu_i q_1^{k-1}\,(1-q_1)^{n-k+1}\,\alpha_i,$$

which can be written as

$$\gamma_i \leq \left(\frac{1-q_0}{q_1}\right)^{k-1} \left(\frac{q_0}{1-q_1}\right)^{n-k+1} = A_{k-1,n}. \tag{7}$$

Hence, sincere voting on alternative 0 (that is, to chose $v_i = 0$ when $s_i = 0$) is optimal if and only if condition (7) is met. Likewise, voting on alternative 1 is optimal if and only if

$$\gamma_i \geq \left(\frac{1-q_0}{q_1}\right)^{k} \left(\frac{q_0}{1-q_1}\right)^{n-k} = A_{k,n} \tag{8}$$

We have proved the following equilibrium counterpart to the optimality result in Theorem 2:

---

[8] Let κ_i^{ω} be the utility of taking the right decision in state $\omega = 0, 1$. Then

$$\kappa_i = \kappa_i^0 \Pr\left[\omega = 0 \mid \mathcal{T} \ \wedge \ s_i = 0\right] + \kappa_i^1 \Pr\left[\omega = 1 \mid \mathcal{T} \ \wedge \ s_i = 0\right]$$

Theorem 3. *Suppose that the signal informativeness condition (3) is met. For any positive $n \in \mathbb{N}$ and $k \in \mathbb{N} \cap [1, n]$, sincere voting under k-majority rule constitutes a Nash equilibrium if and only if*

$$A_{k,n} \leq \gamma_i \leq A_{k-1,n} \qquad \forall i \tag{9}$$

Some remarks are in place. First, if committee members have exactly the same preferences and beliefs, then conditions (9) and (4) are identical. Hence, in this special case, a k-majority rule is optimal (for a committee of given size n) if and only if sincere voting under this rule is a Nash equilibrium. This was first proved by Austen-Smith and Banks (1996, Lemma 2), see also Costinot and Kartik (2006) for more recent findings under the same hypothesis of identical preferences and beliefs.

Secondly, for $k = n = 1$ — the case of a single decision-maker — condition (9) is, as one would expect, identical with the signal informativeness condition (3).

Thirdly, if n is odd and $k = (n+1)/2$ — *simple majority* rule — then (9) and (3) are again identical if $q_0 = q_1$. This is not surprising, since in this knife-edge case when the two signals have identical precision, a tie among all other voters does not affect the odds for one state over the other.[9] Generically, however, q_0 and q_1 are not identical. For instance, the probability of finding evidence against a guilty defendant is arguably not always exactly the same as the probability of finding counter-evidence against an innocent defendant. Suppose, thus, that $q_0 \neq q_1$ and consider majority rule in a committee with an odd number of members. Applied to majority rule with n odd, condition (9) can be written as

$$\frac{1-q_0}{q_1}\left[\frac{q_0(1-q_0)}{(1-q_1)q_1}\right]^m \leq \gamma_i \leq \frac{q_0}{1-q_1}\left[\frac{q_0(1-q_0)}{(1-q_1)q_1}\right]^m \qquad \forall i \tag{10}$$

where $m = (n-1)/2$; half the number of *other* voters. If q_0 and q_1 differ even the slightest, then the factor in square brackets is distinct from unity. As n tends to infinity, this factor either converges to zero, if $q_0 > q_1$, or to plus infinity, if $q_0 < q_1$. Inevitably, one of the two inequalities in condition (9) is thus violated for all n sufficiently large, for any given positive and finite γ_i value. We have proved the following slight generalization of Theorem 1 in Austen-Smith and Banks (1996):[10]

Corollary 2. *Suppose that $q_0 \neq q_1$. For any positive sequence $(\gamma_i)_{i\in\mathbb{N}}$ there exists an $n_0 \in \mathbb{N}$ such that sincere voting is a Nash equilibrium under simple majority rule for no $n \geq n_0$.*

[9] Then $\Pr[\omega = 0 \mid \mathcal{T} \wedge s_i = 0] = \Pr[\omega = 0 \mid s_i = 0]$ and likewise for $\omega = s_i = 1$.

[10] Their result concerns the special case $\alpha_1 = \alpha_2 = ... = \alpha_n = \beta_1 = \beta_2 = ... = \beta_n$.

This result is intuitively plausible. For suppose that state 0 is more likely to give rise to signal value 0 than state 1 is likely to give rise to signal value 1, that is, $q_0 > q_1$. In such a case, signal 0 is less informative than signal 1 in the sense that signal 0 is more likely in state 1 than signal 1 is in state 0. If n is large, a tie among the others is then quite a strong indication of state 1, even if a voter's own signal were 0, since in total there are just about as many signals 0 as signals 1, quite an unlikely event in state 0. Hence, even if I, as a voter, believed that the others vote sincerely, I should vote on alternative 1 in such a case, irrespective of my own signal.

We finally explore the relations between optimality of the voting rule and sincere voting being an equilibrium. It follows immediately from Theorems 2 and 3 that if sincere voting is a Nash equilibrium under a k-majority rule, then this rule is optimal. We already noted that the converse holds if committee members have identical preferences. What if they do not? We proceed to show that if a k-majority rule is optimal, then sincere voting is an equilibrium for sufficiently homogenous committees. Broadly speaking, this should follow from continuity and the discreteness of the set of voting rules. We here identify bounds on preference heterogeneity for the mentioned equivalence between optimality and equilibrium.

To this end, it is useful to first ask which collective decision $x \in \{0,1\}$ individual i then would like to see taken, if i had known the total number N_1 of signals 1 received among all committee members. As shown in the appendix, voter i will prefer decision $x = 1$ over decision $x = 0$ if and only if

$$\frac{\Pr[N_1 \mid \omega = 0]}{\Pr[N_1 \mid \omega = 1]} \leq \gamma_i \tag{11}$$

This can be re-written as $N_1 \geq \lambda_i$, where

$$\lambda_i = \frac{n \ln \frac{q_0}{1-q_1} - \ln \gamma_i}{\ln \frac{q_0}{1-q_1} + \ln \frac{q_1}{1-q_0}} \tag{12}$$

We note that both terms in the denominator are positive and that λ_i decreases with γ_i. Hence, $\lambda_1 \geq \lambda_2 \geq .. . \geq \lambda_n$. For generic parameter values q_0, q_1 and γ_i, the parameters λ_i are not integers. In the sequel, we focus on this generic case. We will call $\lambda_i \in \mathbb{R}$ the *threshold* of i; voter i needs more than λ_i signals 1 to prefer decision 1 (in the jury case, a "guilty" verdict). Write $M_i = \lfloor \lambda_i \rfloor + 1$ for the smallest integer exceeding λ_i. Committee member i thus prefers decision 1 (over decision 0) if and only if the number of signals 1 is at least $M_i \in \mathbb{N}$. We call the committee *homogeneous* if

$$M_i = M_j \qquad \forall i, j \in \{1, ..., n\}, \tag{13}$$

while otherwise the committee will be called *heterogeneous*.[11]

Proposition 1. *If sincere voting is a Nash equilibrium under k-majority rule, then k-majority rule is optimal. Conversely, if k-majority rule is optimal and the committee is homogeneous, then sincere voting under this rule is a Nash equilibrium.*

(Proof in appendix.)

Example 2. *In Example 1 we showed that for $n = 3$, $\bar{\gamma} = 1.35$, $q_0 = 0.8$ and $q_1 = 0.7$ simple majority is the optimal voting rule: $k^* (3) = 2$. Suppose now that the three committee members differ in how they value a mistake of type I (say, acquitting a guilty defendant). Let $\mu_i = 1/2$ and $\beta_i = 1$ for $i = 1, 2, 3$, and let $\alpha_1 = \alpha_2 = 1$ and $\alpha_3 = 2.05$. In other words, committee members 1 and 2 are "doves" while member 3 is a "hawk." With these preferences, $\bar{\gamma} = 1.35$, just as in Example 1, and the informativeness condition (3) is met:*

$$0.286 \simeq \frac{1-q_0}{q_1} \leq \gamma_i \leq \frac{q_0}{1-q_1} \simeq 2.667$$

for $i = 1, 2, 3$. However, sincere voting is not a Nash equilibrium, since committee member 3, the "hawk" needs only 1 positive signal in order to be convinced that $x = 1$ is the right decision:

$$\gamma_3 = 2.05 > A_{1,3} = \left(\frac{1-q_0}{q_1}\right)\left(\frac{q_0}{1-q_1}\right)^2 \simeq 2.03$$

Had there instead been five committee members (say two additional members with $\mu_i = 1/2$, $\alpha_i = 1.35$ and $\beta_i = 1$, for $i = 4$ and 5), then member 3, the hawk, would have an even stronger disincentive to vote sincerely when receiving the signal 0:

$$\gamma_3 = 2.05 > A_{2,3} = \left(\frac{1-q_0}{q_1}\right)^2\left(\frac{q_0}{1-q_1}\right)^3 \simeq 1.55$$

In order to appreciate how binding the homogeneity condition is, let us approximate the integer $\lfloor \lambda_{m+1} \rfloor$ with the real number $\lambda_{m+1} - 1/2$. This approximation is "neutral" in the sense that if $\lfloor \lambda_{m+1} \rfloor = k$ is reported to an outside observer, then $k + 1/2$ is the mid-point of the unit interval to which λ_{m+1} must belong. Using (12) and the fact that the upper and lower bounds for λ_{m+1} differ by one unit

[11] A jury is thus heterogeneous if $\lambda_1 > \lfloor \lambda_{m+1} \rfloor + 1$ or $\lambda_n < \lfloor \lambda_{m+1} \rfloor$ or both.

($N_0 < \lambda_{m+1} < N_0 + 1$), the resulting *approximate* homogeneity condition boils down to:

$$\frac{(1-q_0)(1-q_1)}{q_0 q_1} < \frac{\gamma_i}{\gamma_j} < \frac{q_0 q_1}{(1-q_0)(1-q_1)} \qquad \forall i, j \tag{14}$$

It is easily verified that this condition is implied by our signal informativeness condition (3). The intuition here is that if the signal informativeness condition holds, then the committee as a whole cannot be very heterogeneous. But, as the above example shows, the committee need not be very homogeneous either. We note that the heterogeneity can be larger the higher the signal quality. As $q_0, q_1 \to 1$, the left-hand side of (14) tends to zero while the right-hand side tends to plus infinity.

Remark 2. *Re-consider Theorem 3 and suppose that the signal informativeness condition (3) is met. In order to determine whether sincere voting under k-majority rule, for any given $k \in \mathbb{N} \cap [1, n]$, is a Nash equilibrium or not, it is sufficient to assume that each voter i knows his or her own preferences — whether or not his or her parameter γ_i belongs to the interval $[A_{k,n}, A_{k-1,n}]$. In particular, a committee member does not need to know other committee members' preferences or beliefs. In such an equilibrium, each committee member reasons under the hypothesis that all others vote according to their private signals — their motives for doing so are irrelevant.*

5. Straw vote

It has been suggested (see Coughlan, 2000) that communication before voting can improve the outcome. Coughlan considers the following two-stage voting procedure: in stage one, all committee members simultaneously report their private signal, a zero or a one, to a "center." These reports may be truthful or false. The total counts of reported zeros and ones are made public to the whole committee. In stage two, there is simultaneous voting under some k-majority rule as described above, but now with the total number of reports of each type being common knowledge.

It turns out that under such a straw vote procedure, truthful revelation of one's signal in the communication stage is compatible with subgame perfect equilibrium if voters are identical. In this equilibrium, all committee members have access to the same aggregate information. However, this is but one of many subgame perfect equilibria, many of which are uninformative. For example, even if everyone sends truthful reports, it is optimal to vote on alternative 0 irrespective of the information available, if one expect all others to do likewise. Moreover, as Coughlan (2000) shows, truthful reporting is not compatible with subgame perfection if the committee is heterogeneous in terms of preferences. Members with extreme preferences will not want to truthfully report their information if it "points in the wrong direction" In this section we focus on simple majority rule in a committee with an odd number of members.

Consider, first, a committee consisting of three members with $\gamma_1 \leq \gamma_2 \leq \gamma_3$, and such that the signal informativeness condition (3) is met. For each committee member i, there then exists a minimal number M_i of signals 1 before committee member i prefers decision 1 over decision 0. We then have $M_1 \geq M_2 \geq M_3 > 0$. In other words, committee member 1 is the "dove" and member 3 the "hawk" in the sense of requiring the strongest (weakest) evidence to make decision 1 (say, convict the defendant in a court). However, by (3), all members requires *some* evidence of state 1 — at least one signal 1. Suppose now that we apply the straw vote procedure and that all three committee members truthfully report their signals in the first round. In the second round, they will then have the same information: they all know the number N_1 of signals 1. All four outcomes, $N_1 = 0, 1, 2, 3$, have positive probability. Suppose that $M_1 = 3$ and $M_2 = 1$, that is, the committee member 1 needs three signals 1 in order to prefer decision 1, while the "median voter", member 2, needs only one signal 1 in order to prefer decision 1. If sufficiently dovish, committee member 1 will then not report his or her signal truthfully in the straw vote—the dove will report the signal 0 irrespective of the true signal received. To see this, note that if the other two report truthfully, then they will vote for decision 1 whenever the straw vote results in $N_1 \geq 1$. In other words, if at least one of the two others received the signal 1, the outcome will be decision 1, irrespective of 1's report and subsequent vote. If no committee member received the signal 1 and members 2 and 3 reported truthfully, then member 1 would prefer decision 1, irrespective of his or her own signal. Hence, committee member 1, the dove, will always report signal 0, since either this does not affect the outcome or it sways it away from decision 1 when the evidence for this decision is weak in member 1's eyes. The reason for the failure of the straw vote procedure in this example is the preference heterogeneity in the committee, the big gap between γ_1 and γ_2 resulting in a big gap between M_1 and M_2, here 2.

Secondly, let us consider a committee consisting of an arbitrary odd number n of committee members. Write $n = 2m + 1$, where $m \in \mathbb{N}$. Without loss of generality, suppose that $\gamma_1 \leq \gamma_2 \leq ... \leq \gamma_n$. For each committee member i, let λ_i be as defined in equation (12). Then $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_n$. We will call committee member $i = m+1$ *the median voter* — the unique committee member who has equally many committee members on her "left" as on her "right" on the parameter scale. Let s_i be the signal received by committee member i. Denote by $t_i \in \{0, 1\}$ the *straw vote* of i in stage one of this mechanism and call these votes *reports*. Let the random variable N_1 be the number of reports $t_i = 1$ in the first stage: $N_1 = \sum_{i=1}^{n} t_i$. In the second stage, the committee votes again, now "for real," and the collective decision x is taken according to simple majority rule. The vote cast by individual i at this second stage is denoted v_i. When deciding which vote to cast, voter i knows her own private signal s_i and report t_i, as well as the realization of N_1. Hence, a pure strategy for each committee member i is a pair $(\tau_i, \tilde{\sigma}_i)$, where $\tau_i : \{0,1\} \to \{0,1\}$ assigns a report $t_i = \tau_i(s_i)$ to each signal s_i received, and $\tilde{\sigma}_i : \{0,1\}^2 \times \{0,1,2,...,n\} \to \{0,1\}$ assigns a vote $v_i = \tilde{\sigma}_i(s_i, t_i, N_i)$ to each signal s_i received, report t_i delivered and observed count N_1 of reports $t_j = 1$.

Is truthful reporting compatible with sequential equilibrium? Suppose that all committee members report truthfully in the first stage: $t_i = s_i$ for all i. In stage two, all committee members are then essentially in the same "information set" in stage two, before casting their "real" votes: they all know the total number N_1 of signals 1 among the n signals received. No voter knows exactly who received what signal, except for their own, but this is of no consequence since all voters by assumption receive signals of the same "quality."

Suppose that each committee member votes for his or her preferred decision alternative in stage two, given his or her information. Will each voter i then have an incentive to report truthfully in the first stage? A single voter can change N_1 by only one unit.

Suppose, first, that voter i received the signal $s_i = 1$ and that there were N_0 other signals 1. Then $N_1 = N_0 + 1$ if i will truthfully report $t_i = 1$ while $N_1 = N_0$ if i would falsely report $t_i = 0$. It follows that i's report will affect the final decision x if and only if $N_0 < \lambda_{m+1} < N_0 + 1$, or, equivalently, if and only if $N_0 = \lfloor \lambda_{m+1} \rfloor$.[12] Therefore any committee member i who receives the signal $s_i = 1$ can reason conditionally on this event, namely, that the total number of signals $s_j = 1$ received by the other committee members is exactly $\lfloor \lambda_{m+1} \rfloor$. The probability for this event does not depend on the identity of member i and it does not depend on i's signal or report. This probability is positive whenever $0 \leq \lfloor \lambda_{m+1} \rfloor \leq n$, a condition that can be written as

$$\gamma_{m+1} \leq \left(\frac{q_0}{1 - q_1} \right)^n .$$

But this inequality is implied by the informativeness condition (3) that we already imposed.

[12] The reasoning is based on the assumption that λ_{m+1} does not happen to be an integer.

Likewise, had voter i instead received the signal $s_i = 0$, while the others had still together received N_0 signals 1, then i's report will affect the final decision x if and only if $N_0 - 1 < \lambda_{m+1} < N_0$, or, equivalently, if and only if $N_0 = \lfloor \lambda_{m+1} \rfloor + 1$. Again, this event has positive probability.

It follows that if the committee is homogeneous in the sense defined in connection with Proposition 1, then all committee members will always agree in the second stage about what is the right decision to take. To always report truthfully is then consistent with sequential equilibrium. By contrast, if the committee is heterogeneous, then the strategy to always report truthfully is incompatible with sequential equilibrium. We have established:

Proposition 2. *Suppose that the signal informativeness condition (3) holds and suppose that $\lambda_{m+1} \notin \mathbb{N}$. Truthful reporting in the straw vote is then compatible with sequential equilibrium if and only if the committee is homogeneous.*

The same result for the special case $q_0 = q_1$ can be found in Coughlan (2000), who considers various majoritarian voting rules.

Example 3. *Re-consider Example 2, where $n = 3$, $q_0 = 0.8$, $q_1 = 0.7$, $\mu_i = 1/2$ and $\beta_i = 1$ for $i = 1, 2, 3$, and $\alpha_1 = \alpha_2 = 1$ and $\alpha_3 = 2.05$. In Example 1, we showed that the optimal voting rule was simple majority: $k^*(3) = 2$ and in Example 2 we showed that sincere voting under this rule was incompatible with Nash equilibrium. Would a straw vote help? No, because the "hawk" — committee member 3 — needs only one signals 1 for decision 1, while the two others need 2. More exactly, we have $\gamma_1 = \gamma_2 = 1$ and $\gamma_3 = 2.05$, and thus $\lambda_1 = \lambda_2 \simeq 1.318$ and $\lambda_3 \simeq 0.996$ according to (12), and $M_1 = M_2 = 2$ but $M_3 = 1$. There is no sequential equilibrium in which committee member 1 reports the signal 0 truthfully in the straw vote.*

6. A RANDOMIZED VOTING RULE

Consider first the voting rule according to which each of the n members of a committee casts a vote, a random sample of $h \leq n$ of these votes is drawn and the collective decision is made by way of some k-majority rule applied to this random sample. If each vote has the same positive probability of being sampled, and the sample size h is small enough, then it follows from the above analysis that informative voting will be a Nash equilibrium (take $h = 1$, say).

More precisely, under the signal informativeness condition (3), condition (10) is met for all non-negative integers m sufficiently small (certainly for $m = 0$). Let

m^* be the maximal such integer, and note that m^* depends on the signal quality parameters q_0 and q_1 as well as on the taste-and-belief parameters γ_i, but is independent of m. For any number $n = 2m + 1$ of committee members, let f^* be the randomized voting rule according to which all voters vote simultaneously and the collective decision x is determined by simple majority rule applied to a subset of odd size $h \leq \min\{2m^* + 1, n\}$ of these votes, the subset being drawn at random from among all subsets of size h, with equal probability for each such subset, and where the draw is statistically independent of the state of nature, of the signals and of the votes. In force of Theorem 3 we have:

Corollary 3. *If the signal informativeness condition (3) is met, then informative voting is a Nash equilibrium under the voting rule f^*.*

An evident drawback of this voting rule is that it does not aggregate the private information in an efficient way when n is large, since the sample size h remains bounded by $2m^* + 1$. Hence, while the collective information by the law of large numbers tends asymptotically to the truth as n tends to infinity, just as claimed by Condorcet, the collective decision x under f^* will remain bounded away from being fully informed.

However, there is a straight-forward way to combine this simple randomized majority rule with the usual majority rule and thereby obtain a randomized majority rule that is asymptotically efficient. Instead of *always* letting a randomly selected subset of votes determine the collective decision, suppose that, after everybody has cast their votes, a binary randomization device is employed to determine whether the collective decision x be determined by (i) the simple majority of a random sample of h votes, as described above, or (ii) by the simple majority of all n votes, as described in Section 3. It turns out that under this randomized majority rule, Condorcet's claim can be restored: by choosing the probability for the event (i) carefully, the collective decision will be fully informed with probability one in equilibrium in the limit as n tends to infinity. Indeed, as we will see, this equilibrium is strict and there are no other pure-strategy equilibria and there are no symmetric mixed equilibria either.[13]

For the sake of definiteness and ease of notation, we establish this result for the special case of the simple majority rule applied to a committee with an odd number $n = 2m + 1$ of members (for some $m \in \mathbb{N}$) and random delegation to a single vote,

[13]We conjecture that our equilibrium is unique, but have not yet been able to prove this.

$h = 1$.[14] More precisely, with a pre-specified probability $\varepsilon > 0$ the decision is taken according to a randomly drawn single vote and with probability $1 - \varepsilon$ according to simple majority rule applied to all votes. Denote by f^ε this randomized voting rule, for $0 < \varepsilon < 1$.

We first investigate the condition on the delegation probability ε for sincere voting to be a Nash equilibrium under such a voting rule f^ε. Suppose that committee member i has received the signal $s_i = 0$. Denote by Δu_i^ε the difference in expected utility, for that member, when casting the sincere vote $v_i = 0$ rather than the insincere vote $v_i = 1$. Committee member i thus becomes the "*ex-post* dictator" with probability ε/n, while majority rule is applied to all n votes with probability $1 - \varepsilon$. If another committee member's vote is sampled, then i's vote does not matter. It follows from the above analysis that:

$$\begin{aligned} \Delta u_i^\varepsilon &= \frac{\varepsilon}{n} \frac{(1-\mu_i)\, q_0 \beta_i - \mu_i (1-q_1)\, \alpha_i}{(1-\mu_i)\, q_0 + \mu_i (1-q_1)} + \\ &+ (1-\varepsilon) \binom{2m}{m} \frac{(1-\mu_i)\, q_0^{m+1} (1-q_0)^m \beta_i - \mu_i q_1^m (1-q_1)^{m+1} \alpha_i}{(1-\mu_i)\, q_0 + \mu_i (1-q_1)} \end{aligned} \tag{15}$$

The first term on the right-hand side is non-negative if and only if $\gamma_i \leq q_0/(1-q_1)$. The corresponding condition for signal 1 is $\gamma_i \geq (1-q_0)/q_1$. Both conditions are satisfied under the informativeness condition (3). It follows that then Δu_i^ε is positive for all $\varepsilon < 1$ close enough to 1. Moreover, for ε fixed, it is not difficult to show that the second term on the right-hand side tends to zero as n tends to infinity. Under our preference boundedness condition (2), this holds uniformly for all i (see Appendix). Hence, ε can be small when n is large. Moreover, the incentive for voting sincerely is then strict, so sincere voting is not only a Nash equilibrium but a strict Nash equilibrium. Indeed, this equilibrium is unique. This is the content of the following two results. We will say that the signal informativeness condition (3) is uniformly met if there exists some $\eta < 1$ such that

$$\frac{1-q_0}{\eta q_1} < \gamma_i < \frac{\eta q_0}{1-q_1} \qquad \forall i$$

Theorem 4. *Suppose that the preference boundedness condition (2) holds and that signal informativeness condition (3) is uniformly met. There exist a sequence of positive numbers $\bar{\varepsilon}_n \to 0$ such that for each $n \in \mathbb{N}$ and $\varepsilon \geq \bar{\varepsilon}_n$, under the rule f^ε:*

[14] Our results generalize to any sequence $h_n \leq \max\{2m^* + 1, n\}$, whereby informationally more effient collective decisions are obtained.

(i) sincere voting is a strict Nash equilibrium
(ii) there exists no other Nash equilibrium.

Remark 3. *In the appendix, we prove claim (i) for $\bar{\varepsilon}_n$ decreasing exponentially with n and claim (ii) for $\bar{\varepsilon}_n$ decreasing with n at the rate $1/\sqrt{n}$. We also prove that claim (ii) holds for pure-strategy equilibria and for symmetric mixed-strategy equilibria when $\bar{\varepsilon}_n$ decreases exponentially. However, in order to eliminate asymmetric mixed equilibria, we used an argument that holds only for $\bar{\varepsilon}_n$ decreasing at the rate $1/\sqrt{n}$, though we conjecture that claim (ii) might in fact be true also for exponentially decreasing $\bar{\varepsilon}_n$.*

Let $(\varepsilon_n)_{n\in\mathbb{N}}$ be any sequence of positive numbers tending to zero, such that $\varepsilon_n \geq \bar{\varepsilon}_n$ for all n, where $(\bar{\varepsilon}_n)_{n\in\mathbb{N}}$ satisfies Theorem 4. Let $(f_n)_{n\in\mathbb{N}}$ be the corresponding sequence of randomized majority rules. This sequence of voting rules is asymptotically efficient:

Corollary 4. *Suppose that the preference boundedness condition (2) holds and that the signal informativeness condition (3) is uniformly met. Let $X_n \in \{0,1\}$ be the committee decision under a voting rule f_n such as just described, for each $n \in \mathbb{N}$. Then*

$$\lim_{n\to\infty} \Pr\left[X_n \neq \omega \mid \omega\right] = 0 \qquad \forall \omega \in \Omega$$

(Proof in appendix.)

Example 4. *In Examples 1-3 we studied a committee consisting of three members for which the simple majority rule was optimal but under which sincere voting was not a Nash equilibrium. How large do we need ε to be in order for the randomized majority rule f^ε to induce sincere voting as a (strict) Nash equilibrium? Recall that $q_0 = 0.8$, $q_1 = 0.7$, $\mu_i = 1/2$ and $\beta_i = 1$ for $i = 1,2,3$, and $\alpha_1 = \alpha_2 = 1$ and $\alpha_3 = 2.05$. For any $\varepsilon > 0$ we then have the following expected utility difference between voting 0 and voting 1 upon receiving signal 0, for each committee member i:*

$$\begin{aligned}\Delta u_i^\varepsilon &= \frac{\varepsilon}{3} \cdot \frac{q_0 - (1-q_1)\,\alpha_i}{1+q_0-q_1} + 2\,(1-\varepsilon) \cdot \frac{q_0^2\,(1-q_0) - q_1\,(1-q_1)^2\,\alpha_i}{1+q_0-q_1} \\ &= \frac{\varepsilon}{3} \cdot \frac{0.8 - 0.3\alpha_i}{1.1} + 2\,(1-\varepsilon) \cdot \frac{0.8^2 \cdot 0.2 - 0.7 \cdot 0.3^2\alpha_i}{1.1}\end{aligned}$$

These are affine functions of ε. For member 3, the "hawk," we obtain

$$\Delta u_3^\varepsilon = 0.0581 \cdot \varepsilon - 0.00209.$$

Hence, $\Delta u_3^\varepsilon > 0$ *for all* $\varepsilon \gtrapprox 0.036$. *A "dictatorship delegation probability" of about* 3.6% *(*1.2% *for a given committee member) will thus make sincere voting a strict equilibrium.*

7. A slight preference for sincerity or esteem

Suppose that the committee members have a slight preference for voting according to their personal beliefs. More precisely, in addition to the expected utility from the collective decision, let there be an additional utility from voting sincerely, that is, on the alternative that, given one's preferences, prior and signal is the right decision (has the highest expected utility). Such voters are "rational," it is only that they not only care about the final collective decision but also about the sincerity of their own voting act — of casting their vote in accordance with or against their true belief.

This preference can be intrinsic — a deontological preference for sincerity *per se*. Arguably, many individuals have such preferences. However, a preference for sincerity can also emanate from reputation concerns.[15] Think, for example, of the board of a company or of a central bank.[16] Indeed, in the latter context, the question of increased transparency has been taken up in the public debate, and some central banks (such as the Bank of England and the Sveriges Riksbank) have in recent years introduced transparency rules, whereby individual board members' votes are made public after the decision has been made, and, later on these votes can be evaluated against the backdrop of the economy's performance during the period in question. Similar concerns may apply to other committee and board decisions.

In order to illustrate the effect of such transparency in the context of the present stylized abstract model, suppose now that each individual member of the committee is aware at the moment of voting that there is a positive probability λ that after some time the true state of nature at the time of the decision will be publicly revealed along with the individual's actual vote. Assume, furthermore, that the committee member's esteem (in her own eyes or in others') will increase if his or her vote was right, that is if $v_i = \omega$, while otherwise the esteem falls. We assume that the utility to committee member i of increased esteem from voting right (as compared with voting on the wrong alternative) is positive, $\rho_i > 0$. Given signal 0, for example, the expected utility gain from increased esteem, when voting informatively (that is, on alternative

[15] We are grateful to Torsten Persson for pointing out this possibility.

[16] Clearly, the board of a company or a central bank usually does not face a binary decisions but much more complex decision problems. However, in practice complex deliberations sometimes boil down to a binary decision, to accept or reject a a finalpolicy proposal pitted agains *status quo* or another proposal.

0 instead of on alternative 1) is then

$$\delta_i^0 = \lambda \frac{(1-\mu_i)\, q_0 - \mu_i\,(1-q_1)}{(1-\mu_i)\, q_0 + \mu_i\,(1-q_1)} \cdot \rho_i$$

(see calculations leading up to the informativeness condition (3)), and likewise for the expected utility gain δ_i^1 from increased esteem when voting informatively when the signal is 1.[17] It is easily verified that both δ_i^0 and δ_i^1 are positive if

$$\frac{1-q_0}{q_1} < \frac{\mu_i}{1-\mu_i} < \frac{q_0}{1-q_1}$$

or, equivalently, if the informativeness condition (3) is met for $\alpha_i = \beta_i$.

Maintaining the signal informativeness assumption (3) and assuming that the decision is to be taken by simple majority rule, suppose that each committee member i who receives the signal 0 obtains additional utility $\delta_i^0 > 0$ from voting 0, and likewise if the committee member had received the signal 1. For each committee size $n = 2m+1$, the expected utility difference between voting 0 and 1, for a committee member i who has received the signal 0, now is:

$$\Delta u_i^0 = \delta_i^0 + \binom{2m}{m} \frac{(1-\mu_i)\, q_0^{m+1}\,(1-q_0)^m\, \beta_i - \mu_i q_1^m\,(1-q_1)^{m+1}\, \alpha_i}{(1-\mu_i)\, q_0 + \mu_i\,(1-q_1)} \tag{16}$$

and likewise when the committee member has received the signal 1.

Clearly, sincere voting is a Nash equilibrium for all $\delta_i^0, \delta_i^1 > 0$ sufficiently large. Moreover, if there exists a positive lower bound δ on all δ_i^0 and δ_i^1, then sincere voting is a Nash equilibrium granted n is large enough, since the probability for a tie under sincere voting goes to zero as $n \to +\infty$ and hence the strategic incentive against sincere voting vanishes asymptotically as the size of the committee tends to infinity. In this sense, the negative result in Corollary 2 is not robust. More precisely, the reason for this non-robustness is that the second term in (16) tends to zero as m tends to plus infinity. Hence, no matter how small $\delta_i^0 > 0$ is, Δu_i^0 is positive for m large enough, and likewise for the expected utility difference between voting 1 and 0 for voters who have received the signal 1.

We can firm up these observations as follows. Consider a sequence of committees with ever larger size n, such that the preference boundedness condition (2) holds and the signal informativeness condition (3) is uniformly met. The utility from sincere voting and/or from boosted esteem may depend on the committee size. Arguably, this

[17] $\delta_i^1 = \lambda \frac{\mu q_1 - (1-\mu)(1-q_0)}{(1-\mu)(1-q_0)+\mu q_1} \rho_i$.

utility can plausibly be decreasing in n. In order to allow for this possibility, let $\delta_{i,n}^{0}$ and $\delta_{i,n}^{1}$ be positive for each member i and committee size n. Granted these parameter values are not too close to zero, and do not decrease too fast with n, informative voting is a strict Nash equilibrium, indeed the unique Nash equilibrium for each committee size n. In particular, the whole plethora of uninformative Nash equilibria that exist in the standard voting model in sections 3 and 4 vanishes. Formally:

Theorem 5. *Suppose that the preference boundedness condition (2) holds and the signal informativeness condition (3) is uniformly met. There exist a sequence of positive numbers $\bar{\delta}_n \to 0$ such that for each $n \in \mathbb{N}$, if $\delta_{i,n}^{0}, \delta_{i,n}^{1} \geq \bar{\delta}_n$ for all $i \in \{1, ..., n\}$,*
(i) sincere voting is a strict Nash equilibrium
(ii) there exists no other Nash equilibrium.

(Proof in appendix.)

Condorcet's jury theorem thus holds for rational voters who have a preference for voting according to their own beliefs and/or care about their own reputation. Applying Theorem 1 we immediately obtain:

Corollary 5. *Suppose that the preference boundedness condition (2) holds, that the informativeness condition (3) is uniformly met, and that $\delta_{i,n}^{0}, \delta_{i,n}^{1} \geq \bar{\delta}_n > 0$ for all $n \in \mathbb{N}$ and $i \in \{1, ..., n\}$. Let $X_n(\omega) \in \{0, 1\}$ be the collective decision in pure-strategy Nash equilibrium under majority rule with n voters. Then*

$$\lim_{n \to \infty} \Pr[X_n(\omega) \neq \omega] = 0.$$

The following example gives a numerical illustration of how strong the preference for sincerity and/or esteem needs to be in order to make sincere voting a Nash equilibrium in Examples 1-4.

Example 5. *The diagram below shows the graph of the utility difference Δu_i^0 for committee member $i = 3$, the "hawk" in Examples 1-4, the difference in expected utility from voting sincerely and insincerely, after this member has received the signal 0. On the horizontal axis is m, where the total committee size is $n = 2m + 1$. When adding a utility $\delta_i^0 > 0$ for sincere voting, this curve is lifted by δ_i^0. We see that for a committee size of $n = 3$ only a slight preference for sincerity, $\delta_i^0 \gtrapprox 0.002$, is needed. By contrast, intermediate committee sizes ($5 \leq n \leq 17$) require a stronger preference for sincerity (roughly $\delta_i^0 \simeq 0.03$), while in large committees again only a slight preference for sincerity is required, for large n monotonically vanishing as $n \to \infty$. These* *utility levels for sincerity should be compared with the committee members utility loss from mistakes of types I and II* (2.05 *and* 1*, respectively).*

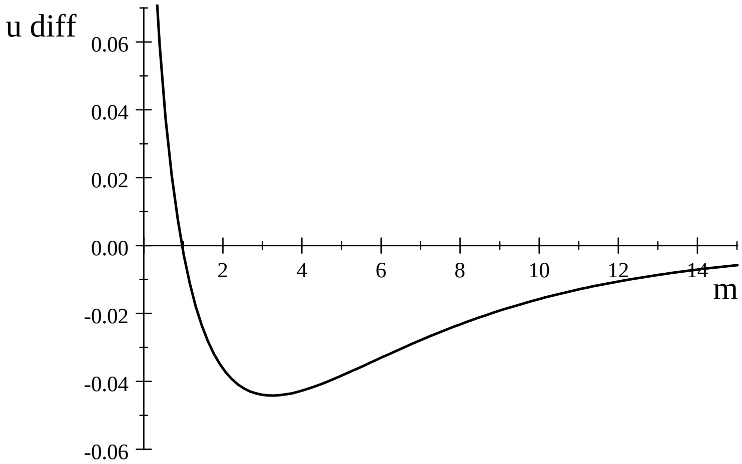

Figure 2: The incentive for sincere voting.

The following example illustrates the possibility of a mixed equilibrium when the preference for sincere voting is not strong enough. In this equilibrium, a "dove" and a "hawk" randomize in an anti-symmetric way.

Example 6. *Consider a committee with tree members with uniform priors* ($\mu_i = 1/2$ *for* $i = 1, 2, 3$)*, both signals equally precise* ($q_0 = q_1 = q$)*, distinct preferences* ($\gamma_1 = 1/c$*,* $\gamma_2 = 1$ *and* $\gamma_3 = c$ *for some* $c > 1$) *and an equally strong preference for sincerity,* ($\delta_i^0 = \delta_i^1 = \delta \geq 0$ *for* $i = 1, 2, 3$)*, such that the signal informativenss condition is strictly met for all committee members. Since the two signals are equally precise, sincere voting is then a strict Nash equilibrium for all* $\delta \geq 0$*. Consider the possibility of an additional equilibrium, a mixed anti-symmetric equilibrium in which (a) voter 1 votes sincerely when receiving signal 0 and votes 1 with probability* x *when receiving signal 1, (b) voter 2 votes sincerely, and (c) voter 3 votes sincerely when receiving signal 1 and votes 0 with probability* x *when receiving signal 0. Does there exist an* x *in the open unit interval for which this constitutes a Nash equilibrium? The critical point is that* x *should render voter 1 indifferent when receiving signal 1 and voter 3 indifferent when receiving signal 0. It is not difficult to show that the anti-symmetry of preferences makes these two indifference conditions coincide, and that they both* *boil down to the following equation:*

$$x = \frac{q\,(c-1)\,(1-q) - \delta}{(c+1)\,(2q-1)\,(1-q)\,q} \tag{17}$$

For $q = 0.8$, $\delta = 0.1$ and $c = 2$, for example, the signal informativeness condition is met and we obtain $x \approx 0.31$. More generally, the equilibrium randomization x is decreasing in δ, the preference for sincerity. For $\delta \geq q\,(c-1)\,(1-q)$, no mixed equilibrium of this sort exists. For example, for $q = 0.8$ and $c = 2$ this is true for all $\delta \geq 0.16$.

In Section 5 we analyzed the potential benefit of a straw vote. We found that, when a straw vote is organized but the committee is not homogeneous in the sense of condition (13), then, according to Proposition 2, truthful reporting is not compatible with sequential equilibrium for the original utilities. The analysis of the present section carries over to the straw-vote setting if committee members obtain additional utilities $\delta_i^0, \delta_i^1 > 0$ from reporting truthfully.

8. CONCLUSION

The above analysis is restricted to a fairly special setting; a committee of equally "competent" members who receive private information of exogenously fixed precision and face a binary collective decision problem without the possibility of abstention. However, we believe that the qualitative conclusions hold more generally.

First, suppose that the committee members are unequally "competent" in the sense that their signals are not equally precise. If these competence differences are known, then weighted majority rule, where more competent voters are given higher weights, may be used to obtain more efficient information aggregation (see Ben-Yashar and Milchtaich (2006)). We believe that our qualitative results concerning equilibrium carry over also to such cases. If differences in competence are not known, then again we believe that our qualitative conclusions hold. For if voters' individual vectors $q^i = (q_0^i, q_1^i)$ are identically and independently distributed according to some fixed probability distribution, and this distribution is not too dispersed, then an application of the law of large numbers will presumably lead to qualitatively the same asymptotic result as we report here.

Secondly, while our hypothesis that committee members know each other's preferences is arguably not so unrealistic in small committees that meet regularly, it is not so realistic in many other situations. Hence, a generalization in this direction is also called for. Note, however, that some of our results would be unaffected if committee members would only have probabilistic beliefs about each others' preferences. For instance, whether or not sincere voting is a Nash equilibrium does not depend on committee members' beliefs or knowledge about each others' preferences. All that matters for a voter is whether or she wants to follow her own signal under the hypothesis that all the others vote sincerely — irrespective of their motives for doing so.

A third direction for generalization, which would be valuable and challenging to explore, concerns the binary nature of both signals and choices. What can be said if the choice is binary but there are more than two signal values (perhaps just three, or a whole continuum)? What if there are more than two choice alternatives?

A forth direction would be to analyze equilibrium outcomes if abstention is an option and/or the number of voters is unknown by the voters. Such aspects may be less relevant for some committees but may play a major role in other committees and certainly in general elections. Krishna and Morgan (2007) undertake an investigation of precisely these two aspects, in a setting where the number of voters is a Poisson distributed random variable and each voter draws a random cost for casting a vote (or going to the polls). The actual number of voters is not observed by the voters and each voter only observes his or her own voting cost. Krishna and Morgan assume that the voters otherwise have identical preferences, that the two states of nature are equally likely and that the two signals are equally precise. The show that sincere voting then is the unique Nash equilibrium under super-majority rules when the expected number of voters is large. Moreover, equilibrium participation rates are such that the outcome is asymptotically efficient. While their model thus is cast more in the mold of general elections, it would be interesting to explore (a) whether our assumptions about preference and belief heterogeneity can be handled in their framework and (b) whether (strategic) abstention in a committee of fixed and know size (without voting costs) can be handled in our framework.

A a fifth and final avenue for further work would be to endogenize voters' signal precision. Before a committee meets, committee members usually make (mostly unobserved) efforts to study relevant information so that they will be well informed at the meeting. However, as is well-known both by practitioners and theorists, this gives rise to a free-rider problem, whereby committee members often tend to under-invest and arrive at the meeting less informed than what would be collectively desirable. For a recent analysis of this phenomenon, see Koriyama and Szentes (2007), who consider a binary effort choice but abstracts from strategic aspects of voting. A synthesis of their approach and ours could potentially lead to interesting new insights.

9. Appendix

We here provide proofs of claims not proved in the main text.

9.1. Theorem 1. Suppose that $\omega = 0$ and consider any positive integer n. The probability that voter i votes $v_i = s_i = 1$, when voting informatively, is $1 - q_0$. Define majority rule by $f(v) = 1$ if $\sum v_i > n/2$**,** $f(v) = 1/2$ if $\sum v_i = n/2$ and otherwise $f(v) = 0$. Under this rule, the probability of a wrong decision in this state is thus

$$\Pr[X_n = 1 \mid \omega = 0] \leq \Pr\left[\frac{1}{n}\sum_{i=1}^{n} s_i \geq \frac{1}{2} \mid \omega = 0\right]$$

Conditional upon $\omega = 0$, the random variables $\{s_i\}_{i=1}^{n}$ are independent, with the same Bernoulli distribution. Hence, according to the Central Limit Theorem (see, for example, Theorem 27.1 in Billingsley, 1995), their average, $\frac{1}{n}\sum_{i=1}^{n} s_i$ (given $\omega = 0$), converges in distribution towards the normal distribution with mean $1 - q_0$ and variance $q_0(1 - q_0)/n$. Since $1 - q_0 < \frac{1}{2}$**:**

$$\lim_{n\to\infty} \Pr\left[\frac{1}{n}\sum_{i=1}^{n} s_i \geq \frac{1}{2} \mid \omega = 0\right] = 0$$

The same argument applies to the state $\omega = 1$.

9.2. Theorem 2. Write $W\left(f^k\right)$ in the following way, where the random variable N_1 is the number of signals 1 received, $U_0 = \sum_{i=1}^{n}(1 - \mu_i)\, u_0^i$ and $U_1 = \sum_{i=1}^{n} \mu_i u_1^i$, two real numbers:

$$\begin{aligned} W\left(f^k\right) &= -\bar{\beta}_n \Pr[x = 1 \mid \omega = 0] - \bar{\alpha}_n \Pr[x = 0 \mid \omega = 1] + U_0 + U_1 \\ &= -\bar{\beta}_n \Pr[N_1 \geq k \mid \omega = 0] - \bar{\alpha}_n \Pr[N_1 < k \mid \omega = 1] + U_0 + U_1 \end{aligned}$$

Hence,

$$W\left(f^{k+1}\right) - W\left(f^k\right) = \bar{\beta}_n \Pr[N_1 = k \mid \omega = 0] - \bar{\alpha}_n \Pr[N_1 = k \mid \omega = 1]$$

and thus

$$\begin{aligned} W\left(f^{k+1}\right) \leq W\left(f^k\right) &\iff \bar{\gamma} \geq \frac{\Pr[N_1 = k \mid \omega = 0]}{\Pr[N_1 = k \mid \omega = 1]} \\ &\iff \bar{\gamma} \geq \frac{(1 - q_0)^k\, q_0^{n-k}}{q_1^k\,(1 - q_1)^{n-k}} = A_{k,n} \end{aligned}$$

Likewise:

$$\begin{aligned} W\left(f^{k-1}\right) \leq W\left(f^{k}\right) \quad &\Longleftrightarrow \quad \bar{\gamma} \leq \frac{\Pr\left[N_1 = k-1 \mid \omega = 0\right]}{\Pr\left[N_1 = k-1 \mid \omega = 1\right]} \\ &\Longleftrightarrow \quad \bar{\gamma} \leq \frac{\left(1-q_0\right)^{k-1} q_0^{n-k+1}}{q_1^{k-1}\left(1-q_1\right)^{n-k+1}} = A_{k-1,n} \end{aligned}$$

Since $A_{k,n}$ is decreasing in k, $W\left(f^{k}\right) \geq W\left(f^{h}\right)$ for all $h = k+1, k+2, ..., n$ if and only if $\bar{\gamma} \geq A_{k,n}$. Likewise, $W\left(f^{k}\right) \geq W\left(f^{h}\right)$ for all $h = k-1, k-2, ..., 1$ if and only if $\bar{\gamma} \leq A_{k-1,n}$. Hence, as k increases from 1 to n, $W\left(f^{k}\right)$ reaches its maximum value either at a unique k or (non-generically) at two adjacent values, $k-1$ and k. As noted in footnote 6, $k=0$ and $k=n+1$ are never optimal.

9.3. Corollary 1. Condition (4) is equivalent with

$$\left[\frac{\left(1-q_0\right)\left(1-q_1\right)}{q_0 q_1}\right]^{k}\left(\frac{q_0}{1-q_1}\right)^{n} \leq \bar{\gamma}_n \leq \left[\frac{\left(1-q_0\right)\left(1-q_1\right)}{q_0 q_1}\right]^{k}\left(\frac{q_1}{1-q_0}\right)\left(\frac{q_0}{1-q_1}\right)^{n+1}$$

or

$$\left(\frac{q_0}{1-q_1}\right)^{n} \leq \bar{\gamma}_n \cdot \left[\frac{q_0 q_1}{\left(1-q_0\right)\left(1-q_1\right)}\right]^{k} \leq \left(\frac{q_1}{1-q_0}\right)\left(\frac{q_0}{1-q_1}\right)^{n+1}$$

Taking logarithms and dividing through with n, we obtain

$$\begin{aligned} \ln\left(\frac{q_0}{1-q_1}\right) \quad &\leq \quad \frac{1}{n}\ln\bar{\gamma}_n + \frac{k}{n}\ln\left[\frac{q_0 q_1}{\left(1-q_0\right)\left(1-q_1\right)}\right] \\ &\leq \quad \frac{1}{n}\ln\left(\frac{q_1}{1-q_0}\right) + \left(1+\frac{1}{n}\right)\ln\left(\frac{q_0}{1-q_1}\right) \end{aligned}$$

As $n \to \infty$, the upper bound converges to the lower bound, $\ln\left(\frac{q_0}{1-q_1}\right)$, and $\frac{1}{n}\ln\bar{\gamma}_n$ tends to zero since,from our preference boundedness condition, $\bar{\gamma}_n \in \left[\gamma_{\min}, \gamma_{\max}\right]$ for all $n \in \mathbb{N}$. This establishes the equality in (5).

In order to establish the claimed inequalities, let

$$B = \frac{\ln\left(\frac{q_0}{1-q_1}\right)}{\ln\left(\frac{q_0}{1-q_1}\right) + \ln\left(\frac{q_1}{1-q_0}\right)}.$$

Suppose first that $q_1 \leq q_0$. Then $\frac{q_1}{1-q_0} \leq \frac{q_0}{1-q_1}$, from which we deduce that $B \geq 1/2$ and $1-q_0 \leq B$. To obtain the second claimed inequality, $B \leq q_1$, note that this can be re-written, after some manipulation, as

$$q_1 \ln(1-q_0) + (1-q_1)\ln q_0 \leq q_1 \ln q_1 + (1-q_1)\ln(1-q_1).$$

The right-hand side is independent of q_0, while the left-hand side is decreasing in q_0. Thus, the claimed inequality $B \leq q_1$ holds for all $q_0 \in [q_1, 1]$ if and only if it holds for $q_0 = q_1$. Writing the inequality for that special case, one obtains

$$q_1 \ln(1-q_1) + (1-q_1)\ln q_1 \leq q_1 \ln q_1 + (1-q_1)\ln(1-q_1),$$

or, equivalently,

$$(2q_1 - 1)\ln(1-q_1) \leq (2q_1 - 1)\ln q_1,$$

an inequality which clearly holds since $q_1 \geq 1/2$.

Now suppose that $q_1 \leq q_0$. Then the above reasoning (switching q_0 and q_1) gives us $1 - q_1 \leq 1 - B \leq q_0$, which is equivalent with the claimed inequality $1 - q_0 \leq B \leq q_1$.

9.4. Proposition 1. Suppose that sincere voting is an equilibrium under k-majority rule, then, by Theorem 3, $A_{k,n} \leq \gamma_i \leq A_{k-1,n}$ for all i. Hence

$$(1-\mu_i)\,\beta_i A_{k,n} \leq \mu_i \alpha_i \leq (1-\mu_i)\,\beta_i A_{k-1,n} \qquad \forall i$$

so $\bar{\beta}_n A_{k,n} \leq \bar{\alpha}_n \leq \bar{\beta}_n A_{k-1,n}$ or, equivalently, $A_{k,n} \leq \bar{\gamma}_n \leq A_{k-1,n}$. Thus, by Theorem 2, the k-majority rule is optimal.

Conversely, suppose that the committee is homogeneous. There exists an integer M such that $M - 1 \leq \lambda_i \leq M$ for all i. By definition (12) of λ_i, this is equivalent to

$$\left[\frac{q_0}{1-q_1}\right]^{n-M}\left[\frac{1-q_0}{q_1}\right]^{M-1} \leq \gamma_i \leq \left[\frac{1-q_0}{q_1}\right]^{M-1}\left[\frac{q_0}{1-q_1}\right]^{n-M} \qquad \forall i \tag{18}$$

This implies the same inequality for $\bar{\gamma}_n$ and thus k-majority rule is optimal for $k = M$, by Theorem 2. But since (18) holds, sincere voting is an equilibrium, by Theorem 3.

9.5. Claim (i) in Theorem 4. To see that sincere voting under f^ε is a strict Nash equilibrium, first note that $\Delta u_i^\varepsilon > 0$ if and only if

$$\frac{\varepsilon}{1-\varepsilon} > \frac{2m+1}{B_i} \cdot \binom{2m}{m}\left[\alpha_i \mu_i q_1^m (1-q_1)^{m+1} - \beta_i (1-\mu_i)\, q_0^{m+1} (1-q_0)^m\right] \tag{19}$$

where the factor $B_i = \beta_i (1-\mu_i)\, q_0 - \alpha_i \mu_i (1-q_1)$ is positive by (3). By Stirling's formula,

$$\binom{2m}{m} = \frac{(2m)!}{(m!)^2} = \frac{4^m}{\sqrt{\pi m}}(1 + o(m))$$

so the right-hand side of (19) is approximated by

$$\begin{aligned}
&(1+o(m))\cdot\frac{4^m}{B_i(2m+1)\sqrt{\pi m}}\cdot\left[\alpha_i\mu_i q_1^m\left(1-q_1\right)^{m+1}-\beta_i\left(1-\mu_i\right)q_0^{m+1}\left(1-q_0\right)^m\right]\\
\leq\ &(1+o(m))\cdot\frac{2\alpha_i\mu_i\left(1-q_1\right)}{B_i\sqrt{\pi}}[4q_1(1-q_1)]^m\sqrt{m}\leq(1+o(m))\cdot\frac{C_i}{B_i}\cdot a^m\sqrt{m}
\end{aligned}$$

where $C_i = 2\alpha_i\mu_i\left(1-q_1\right)/\sqrt{\pi}$ and $a = 4q_1(1-q_1) < 1$. Hence, (19) is met if

$$\frac{\varepsilon}{1-\varepsilon} > \frac{C_i}{B_i}(1+o(m))a^m\sqrt{m}$$

A sufficient condition for this to hold is that

$$\varepsilon > \frac{C_i}{B_i}(1+o(m))a^m\sqrt{m} \tag{20}$$

The preference boundedness condition (2) together with the hypothesis that the signal informativeness condition is uniformly met implies that C_i/B_i is uniformly bounded: there exists a $D\in\mathbb{R}$ such that $C_i/B_i < D$ for all i.[18] Let $\varepsilon = b^m$ with $a < b < 1$. Then $\varepsilon \to 0$ as $m \to +\infty$. Moreover, since

$$\left(\frac{b}{a}\right)^m \frac{1}{\sqrt{m}} \to +\infty \quad \text{as} \quad m\to\infty,$$

(20) holds for all m large enough, irrespective of how large D is.

The same reasoning applies to the expected utility upon receiving the signal $s_i = 1$. This proves claim (i) for $\varepsilon = b^m$, for any b such that

$$\max\{4q_0(1-q_0), 4q_1(1-q_1)\} < \quad b \quad < \quad 1$$

where we note that lower bound indeed is less than 1 since $q_0, q_1 > 1/2$.

9.6. Corollary 4. Suppose first that $\omega = 0$ and consider informative voting under f^n, for $n = 2m+1 \in \mathbb{N}$ fixed. The probability that committee member i votes $s_i = 1$ is, by definition $1 - q_0$. If the collective decision is taken by simple majority rule

[18] To see this, note that $C_i/B_i \leq D$ iff

$$\frac{1}{\gamma_i}\cdot\frac{q_0}{1-q_1} - 1 \geq \frac{1}{D}$$

and let $\eta = D/\left(D+1\right)$.

applied to all n votes, the probability of a wrong decision, $X_n = 1$, is some number Q_n. So the probability of a wrong decision, given $\omega = 0$, is

$$\Pr\left[X_n = 1 \mid \omega = 0\right] = \varepsilon_n(1 - q_0) + (1 - \varepsilon_n)Q_n$$

The probability of a wrong decision in state $\omega = 0$ thus tends to 0 if $Q_n \to 0$ as $n \to \infty$ since $\varepsilon_n \to 0$. It thus remains to prove that $Q_n \to 0$. We proceed just as in the proof of Condorcet's jury theorem. First note that, since n is odd:

$$Q_n = \Pr\left[\sum_{i=1}^{n} s_i > \frac{n}{2} \mid \omega = 0\right]$$

Conditional upon $\omega = 0$, the signals s_i are independent, with the same Bernoulli distribution. Hence, according to the Central Limit Theorem, $\frac{1}{n}\sum_{i=1}^{n} s_i$, given $\omega = 0$, converges in distribution to the normal distribution with mean $1 - q_0$ and variance $q_0(1 - q_0)/n$. Since $q_0 > \frac{1}{2}$:

$$\lim_{m\to\infty} \Pr\left[\frac{1}{n}\sum_{i=1}^{n} s_i > \frac{1}{2} \mid \omega = 0\right] = 0.$$

The same argument applies to the case $\omega = 1$.

9.7. Claim (i) in Theorem 5. In order to establish that informative voting constitutes a strict Nash equilibrium, consider, first, a voter who has received the signal 0. Under majority rule applied to $n = 2m + 1$ voters, the expected utility difference between voting 0 and 1 is given by (6). Focusing on large n and applying Stirling's formula,

$$m! = \sqrt{2\pi m} \cdot (m/e)^m \cdot (1 + o(m)),$$

we have

$$\binom{2m}{m} = \frac{(2m)!}{(m!)^2} = \frac{4^m}{\sqrt{\pi m}}(1 + o(m))$$

and obtain

$$\begin{aligned}
\lim_{m\to\infty} \Delta u_i^0 &= \delta_i^0 + \lim_{m\to\infty} \frac{4^m}{\sqrt{\pi m}} \cdot \frac{\beta_i\left(1-\mu_i\right) q_0\left[q_0\left(1-q_0\right)\right]^m - \alpha_i\mu_i\left(1-q_1\right)\left[q_1\left(1-q_1\right)\right]^m}{\left(1-\mu_i\right) q_0 + \mu_i\left(1-q_1\right)} \\
&\geq \delta_i^0 - \lim_{m\to\infty} \frac{1}{\sqrt{\pi m}} \cdot \frac{\alpha_i\mu_i\left(1-q_1\right)\left[4q_1\left(1-q_1\right)\right]^m}{\left(1-\mu_i\right) q_0 + \mu_i\left(1-q_1\right)} \\
&= \delta_i^0 - \lim_{m\to\infty} \frac{1}{\sqrt{\pi m}} \cdot \frac{\alpha_i\mu_i\left(1-q_1\right)}{\left(1-\mu_i\right) q_0 + \mu_i\left(1-q_1\right)} \cdot a^m = \delta_i^0 - 0 = \delta_i^0 \geq \delta > 0,
\end{aligned}$$

because $1/2 < q_1 < 1$ implies that $a = 4q_1(1 - q_1) < 1$. The same holds for a voter who has received the signal 1. Claim (i) is thus obtained in much the same way as claim (i) in Theorem 4, namely, for a sequence of δ-values decreasing in n at the rate b^m, for $m = (n-1)/2$, where $\max\{4q_0(1-q_0), 4q_1(1-q_1)\} < b < 1$.

9.8. Claim (ii) in Theorems 4 and 5, pure strategies. The line of reasoning is the same for Theorems 4 and 5, so we thus treat both cases in this section. For any non-negative integer m, the base-line game, that is, simple majority rule applied to a committee of size $n = 2m + 1$, will be denoted $G(m)$. Let $G_\delta(m)$ denote the game when majority rule is applied to a committee of size $n = 2m + 1$ in which each committee member obtains an additional utility, a "sincerity bonus," of δ from voting sincerely in each state of the world, and where $\delta > 0$ is such that sincere voting is a Nash equilibrium. (The following argument for uniqueness goes through also in the more general case when $\delta > 0$ instead is a lower bound on bonuses δ_i^ω). Finally, let $\Phi_\varepsilon(m)$ denote the game in which the randomized majority rule f^ε is applied to a committee of size $n = 2m + 1$ and where $\varepsilon > 0$ is such that sincere voting is a Nash equilibrium.

In all these games, a *pure strategy* for any voter i is a mapping from i's signal to i's vote:

$$\sigma_i : \{0, 1\} \to \{0, 1\} \tag{21}$$

We will write $\sigma_i(s_i) = v_i$ when i's signal is s_i and vote is v_i. Thus each voter has four pure strategies:

- The *informative* strategy, σ^+ defined by: $\sigma^+(0) = 0$ and $\sigma^+(1) = 1$
- The *reversed* strategy, σ^- defined by: $\sigma^-(0) = 1$ and $\sigma^-(1) = 0$
- The *constant*-0 strategy, σ^0 defined by: $\sigma^0(0) = 0$ and $\sigma^0(1) = 0$
- The *constant*-1 strategy, σ^1 defined by: $\sigma^1(0) = 1$ and $\sigma^1(1) = 1$

Let the strategies of players $j = 1, ..., 2m$ be fixed and consider voter $i = 2m + 1$.

Denote by $\mathcal{T}$ the event of a tie among all other votes. Since signals are independent, conditionally on the state, we have:

$$\Pr[\mathcal{T} \wedge (s_{2m+1} = 0) \mid \omega] = \Pr[\mathcal{T} \mid \omega] \cdot \Pr[s_{2m+1} = 0 \mid \omega]$$

for all states ω. Let $\pi_0 = \Pr[\mathcal{T} \mid \omega = 0]$ and $\pi_1 = \Pr[\mathcal{T} \mid \omega = 1]$. For player $i = 2m + 1$, all strategies have the same payoff consequences unless the event $\mathcal{T}$ realizes.

Therefore, only the following four events have to be considered when evaluating i's strategy choice:

event	probability	σ^+	σ^-	σ^0	σ^1
$\mathcal{T} \wedge (s_{2m+1} = 0) \wedge (\omega = 0)$	$\pi_0 q_0 (1 - \mu_i)$	0	$-\beta_i$	0	$-\beta_i$
$\mathcal{T} \wedge (s_{2m+1} = 1) \wedge (\omega = 0)$	$\pi_0 (1 - q_0)(1 - \mu_i)$	$-\beta_i$	0	0	$-\beta_i$
$\mathcal{T} \wedge (s_{2m+1} = 0) \wedge (\omega = 1)$	$\pi_1 (1 - q_1) \mu_i$	$-\alpha_i$	0	$-\alpha_i$	0
$\mathcal{T} \wedge (s_{2m+1} = 1) \wedge (\omega = 1)$	$\pi_1 q_1 \mu_i$	0	$-\alpha_i$	$-\alpha_i$	0

The difference in expected utility for voter $i = 2m + 1$ between the informative and reversed strategies is in $G_\delta(m)$:

$$\mathbb{E}\left[u_i(\sigma^+)\right] - \mathbb{E}\left[u_i(\sigma^-)\right] = \pi_0(2q_0 - 1)(1 - \mu_i)\beta_i + \pi_1(2q_1 - 1)\mu_i\alpha_i + \delta.$$

By assumption, $\alpha_i, \beta_i > 0$ and $q_0, q_1 > 1/2$. Hence:

$$\mathbb{E}\left[u_i(\sigma^+)\right] - \mathbb{E}\left[u_i(\sigma^-)\right] \geq \delta,$$

where $\delta \geq 0$. Moreover, unless π_0 and π_1 are both equal to 0, $\mathbb{E}\left[u_i(\sigma^+)\right] - \mathbb{E}\left[u_i(\sigma^-)\right] > \delta$. However, there exist strategy profiles for which $\pi_0 = \pi_1 = 0$.

This argument shows that, in the base-line game $G(m)$:

- The informative strategy σ^+ weakly dominates the reversed strategy σ^-.
- If a strategy profile is such that player i has a positive probability of being pivotal, then the reversed strategy σ^- is not a best response for player i.

It is easy to see that even stronger conclusions hold in all the variants of the base-line game studied in this paper. With ethical voters, $\delta > 0$, we have $\mathbb{E}\left[u_i(\sigma^+)\right] > \mathbb{E}\left[u_i(\sigma^-)\right]$, and with random delegation, the reversed strategy can be a best response only if the player is pivotal in no sample. This is impossible when $h = 1$ (delegation to a single vote). Hence:

Lemma 1. *If $\delta, \varepsilon > 0$, then σ^- is used with probability zero in every Nash equilibrium of $G_\delta(m)$ and in every Nash equilibrium of $\Phi_\varepsilon(m)$.*

Our next lemma further restricts the set of potential equilibria under majority rule with ethical voters and in randomized dictatorship.

Lemma 2. *If $\delta, \varepsilon > 0$, then there exists no pure Nash equilibrium of $G_\delta(m)$ or $\Phi_\varepsilon(m)$ in which both σ^0 and σ^1 are used.*

Proof: Consider a pure equilibrium in which both strategies σ^0 and σ^1 are used. Let n^+, n^0 and n^1 denote the number of players who respectively use the strategies σ^+, σ^0 and σ^1, with $n^+ + n^0 + n^1 = n = 2m+1$. One player, say i, plays the pure strategy σ^0 and another player, say j, plays σ^1. Denote by V^0 the (random) number of zero votes among the n^+ voters who vote informatively and let $V^1 = n^+ - V^0$. Since player i herself votes 0, i is pivotal if and only if $V^0 + n^0 - 1 = V^1 + n^1$. So the event that i is pivotal is:

$$\mathcal{T}_i = \left\{V^0 + n^0 - 1 = V^1 + n^1\right\} = \left\{V^0 = d\right\},$$

where $d = m - n^0 + 1$. Likewise for voter j:

$$\mathcal{T}_j = \left\{V^0 + n^0 = V^1 + n^1 - 1\right\} = \left\{V^0 = d - 1\right\}.$$

Let

$$\begin{aligned} \pi_0^i &= \Pr\left[\mathcal{T}_i \mid \omega = 0\right], \; \pi_1^i = \Pr\left[\mathcal{T}_i \mid \omega = 1\right] \\ \pi_0^j &= \Pr\left[\mathcal{T}_j \mid \omega = 0\right], \; \pi_1^j = \Pr\left[\mathcal{T}_j \mid \omega = 1\right]. \end{aligned}$$

We now look for possible strategic deviations. For player i, the expected utilities for the three strategies in the base-line game $G(m)$ are of the form

$$\begin{aligned} \mathbb{E}\left[u_i(\sigma^+)\right] &= \text{constant} - \pi_0^i(1-q_0)\left(1-\mu_i\right)\beta_i - \pi_1^i(1-q_1)\mu_i\alpha_i \\ \mathbb{E}\left[u_i(\sigma^0)\right] &= \text{constant} - \pi_1^i\mu_i\alpha_i \\ \mathbb{E}\left[u_i(\sigma^1)\right] &= \text{constant} - \pi_0^i\left(1-\mu_i\right)\beta_i, \end{aligned}$$

and similar expressions hold for player j.

In an equilibrium of $G(m)$, i is better off playing σ^0 than σ^+, and j is better off playing σ^1 than σ^+, so:

$$\begin{aligned} \mathbb{E}\left[u_i(\sigma^0)\right] - \mathbb{E}\left[u_i(\sigma^+)\right] &= -\pi_1^i\mu_i\alpha_i + \pi_0^i(1-q_0)\left(1-\mu_i\right)\beta_i + \pi_1^i(1-q_1)\mu_i\alpha_i \geq 0 \quad (22) \\ \mathbb{E}\left[u_j(\sigma^1)\right] - \mathbb{E}\left[u_j(\sigma^+)\right] &= -\pi_0^j\left(1-\mu_i\right)\beta_i + \pi_0^j(1-q_0)\left(1-\mu_i\right)\beta_i + \pi_1^j(1-q_1)\mu_i\alpha_i \geq 0 \end{aligned}$$

where

$$\begin{aligned} \pi_0^i &= \binom{n^+}{d} q_0^d (1-q_0)^{n^+-d} \\ \pi_1^i &= \binom{n^+}{d} q_1^{n^+-d} (1-q_1)^d \\ \pi_0^j &= \binom{n^+}{d-1} q_0^{d-1} (1-q_0)^{n^+-d+1} \\ \pi_1^j &= \binom{n^+}{d-1} q_1^{n^+-d+1} (1-q_1)^{d-1}. \end{aligned}$$

when the factorials $\binom{n^+}{d}$, $\binom{n^+}{d-1}$ are well-defined. If $d < 0$ or $d > n^+$ then $\pi_0^i = \pi_1^i = 0$ and if $d-1 < 0$ or $d-1 > n^+$ then $\pi_0^j = \pi_1^j = 0$.

In a game $G_\delta(m)$:

$$\begin{aligned} \mathbb{E}\left[u_i^\delta(\sigma^+)\right] &= \mathbb{E}\left[u_i(\sigma^+)\right] + \delta \\ \mathbb{E}\left[u_i^\delta(\sigma^0)\right] &= \mathbb{E}\left[u_i(\sigma^0)\right] + \delta \Pr\left[s_i = 0\right] \\ \mathbb{E}\left[u_i^\delta(\sigma^1)\right] &= \mathbb{E}\left[u_i(\sigma^1)\right] + \delta \Pr\left[s_i = 1\right] \end{aligned} \tag{23}$$

and (22) for σ^0 for player i becomes:

$$\mathbb{E}\left[u_i^\delta(\sigma^0)\right] - \mathbb{E}\left[u_i^\delta(\sigma^+)\right] = \mathbb{E}\left[u_i(\sigma^0)\right] - \mathbb{E}\left[u_i(\sigma^+)\right] + \delta\left(\Pr\left[s_i = 0\right] - 1\right) \geq 0.$$

Since $\Pr\left[s_i = 0\right] < 1$ this implies a strict inequality in (22) in the case $\delta > 0$. The same thing holds for σ^1 so that both inequalities in (22) are strict in the case $\delta > 0$.

In a game $\Phi_\varepsilon(m)$ with $0 < \varepsilon < 1$:

$$\begin{aligned} \mathbb{E}\left[u_i^\varepsilon(\sigma^+)\right] &= (1 - \frac{\varepsilon}{n})\mathbb{E}\left[u_i(\sigma^+)\right] - \frac{\varepsilon}{n}\left(\beta_i\left(1-\mu_i\right)\left(1-q_0\right) + \alpha_i\mu_i(1-q_1)\right) \\ \mathbb{E}\left[u_i^\varepsilon(\sigma^0)\right] &= (1 - \frac{\varepsilon}{n})\mathbb{E}\left[u_i(\sigma^0)\right] - \frac{\varepsilon}{n}\alpha_i\mu_i \\ \mathbb{E}\left[u_i^\varepsilon(\sigma^1)\right] &= (1 - \frac{\varepsilon}{n})\mathbb{E}\left[u_i(\sigma^1)\right] - \frac{\varepsilon}{n}\beta_i\left(1-\mu_i\right) \end{aligned} \tag{24}$$

and (22) for σ^0 becomes:

$$\begin{aligned} \mathbb{E}\left[u_i^\varepsilon(\sigma^0)\right] - \mathbb{E}\left[u_i^\varepsilon(\sigma^+)\right] &= (1 - \frac{\varepsilon}{n})\left(\mathbb{E}\left[u_i(\sigma^0)\right] - \mathbb{E}\left[u_i(\sigma^+)\right]\right) \\ + \frac{\varepsilon}{n}\left(\beta_i\left(1-\mu_i\right)\left(1-q_0\right) - \alpha_i\mu_i q_1\right) &\geq 0, \end{aligned}$$

and likewise for σ^1. By hypothesis, the informativeness condition (3) is met, so we again obtain that both inequalities in (22) are strict in the case $\varepsilon > 0$.[19]

It follows that both in $G_\delta(m)$ and $\Phi_\varepsilon(m)$, (22) becomes:

$$\begin{aligned} \pi_1^i q_1 \gamma_i &< \pi_0^i(1-q_0) \\ \pi_0^j q_0 &< \pi_1^j(1-q_1)\gamma_i \end{aligned} \tag{25}$$

In the case $1 \leq d \leq n^+$ the formulae given above for the probabilities π give:

$$\begin{aligned} q_1^{n^+-d+1}(1-q_1)^d\gamma_i &< q_0^d(1-q_0)^{n^+-d+1} \\ q_0^d(1-q_0)^{n^+-d+1} &< q_1^{n^+-d+1}(1-q_1)^d\gamma_i, \end{aligned}$$

[19] It should be clear at this point that the same reasonning is valid if we simultaneously consider delegation ε and ethical bonus δ.

a contradiction. If $d \leq 0$ or $d \geq n^+ + 1$, then $\pi_0^j = \pi_1^j$ and thus the second strict inequality in (25) is a contradiction. **QED**

Having established that there is no pure-strategy equilibrium in which both constant strategies are used, we finally show that there is no pure-strategy equilibrium in which any one of them is used. This establishes our uniqueness claims.

Formally:

Lemma 3. *Under the conditions of the theorem, there exists an* $m^0 \in \mathbb{N}$ *such that, for all* $m > m^0$, $\varepsilon \geq b^m$ *and* $\delta > 0$, *neither* σ^0 *nor* σ^1 *is used in any pure Nash equilibrium of* $G_\delta(m)$ *and* $\Phi_\varepsilon(m)$.

Proof: Suppose that $n^1 = 0$, so that only σ^0 and σ^+ may be used, with

$$n^+ + n^0 = n = 2m + 1.$$

If $n^0 \geq m + 2$ then a σ^0-strategist cannot be pivotal in the base-line game, so she strictly improves her payoff by deviating to σ^+.

If $0 < n^0 \leq m + 1$ (that is $d \geq 0$), then σ^0 and σ^+ are both used. Let i be a σ^0-strategist. In the base-line game,

$$\begin{aligned}
\mathbb{E}\left[u_i(\sigma^0)\right] - \mathbb{E}\left[u_i(\sigma^+)\right] &= \pi_0^i (1 - q_0)(1 - \mu_i)\,\beta_i - \pi_1^i q_1 \mu_i \alpha_i \\
&= (1 - \mu_i)\,\beta_i \binom{n^+}{d} \left[q_0^d (1 - q_0)^{n^+ - d + 1} - \gamma_i q_1^{n^+ - d + 1} (1 - q_1)^d\right] \\
&= (1 - \mu_i)\,\beta_i \binom{m + d}{d} \left[q_0^d (1 - q_0)^{m+1} - \gamma_i q_1^{m+1} (1 - q_1)^d\right]
\end{aligned}$$

write

$$A_{m,d} = \binom{m + d}{d} \left[q_0^d (1 - q_0)^{m+1} - \gamma_i q_1^{m+1} (1 - q_1)^d\right]$$

In $G_\delta(m)$ or $\Phi_\varepsilon(m)$, for $\delta, \varepsilon > 0$, there is an additional positive "bonus" to the sincere strategy (equations 23, 24) and this bonus does not depend on d.

We now establish the following

Claim. Given m and $0 \leq d \leq m$: if $A_{m,d} > 0$ then $A_{m,d} \leq A_{m,m}$.

To prove this claim, remember that

$$\binom{m + d}{d} = \frac{m + d}{d} \binom{m + d - 1}{d - 1}$$

and write:

$$\frac{A_{m,d}}{A_{m,d-1}} = \frac{m+d}{d} \cdot q_0 \cdot \frac{q_0^{d-1}(1-q_0)^{m+1} - \gamma_i \frac{1-q_1}{q_0} q_1^{m+1}(1-q_1)^{d-1}}{q_0^{d-1}(1-q_0)^{m+1} - \gamma_i q_1^{m+1}(1-q_1)^{d-1}}$$

If $A_{m,d-1} > 0$, the denominator $q_0^{d-1}(1-q_0)^{m+1} - \gamma_i q_1^{m+1}(1-q_1)^{d-1}$ is positive and then, since $\frac{1-q_1}{q_0} < 1$, the numerator $q_0^{d-1}(1-q_0)^{m+1} - \gamma_i \frac{1-q_1}{q_0} q_1^{m+1}(1-q_1)^{d-1}$ is larger than the denominator, so that

$$\frac{A_{m,d}}{A_{m,d-1}} > \frac{m+d}{d} \cdot q_0$$

But $q_0 > 1/2$ and $m + d \geq 2d$. Therefore $A_{m,d} > A_{m,d-1}$ and the claim is proved.

It follows that, in $G_\delta(m)$, if $\mathbb{E}u_i^\delta(\sigma^0) \geq \mathbb{E}u_i^\delta(\sigma^+)$ for some $d < m$, then $\mathbb{E}u_i^\delta(\sigma^0) > \mathbb{E}u_i^\delta(\sigma^+)$ for $d = m$. Clearly the same thing is true if σ^0 is replaced by σ^1.

The case $d = m$ is when all other voters vote informatively. But we know that, for m large enough, informative voting is a Nash equilibrium, which implies

$$\mathbb{E}u_i^\delta(\sigma^0), \mathbb{E}u_i^\delta(\sigma^1) \leq \mathbb{E}u_i^\delta(\sigma^+)$$

It must therefore be the case that for no d, $\mathbb{E}u_i^\delta(\sigma^0) \geq \mathbb{E}u_i^\delta(\sigma^+)$ nor $\mathbb{E}u_i^\delta(\sigma^1) \geq \mathbb{E}u_i^\delta(\sigma^+)$. This proves the lemma for $G_\delta(m)$. The same reasoning applies to $\Phi_\varepsilon(m)$.

9.9. Claim (ii) in Theorems 4 and 5, general case. We here show that all equilibria are pure, under the hypotheses of Theorems 4 and 5, respectively. Consider voter strategies $\sigma_i : \{0,1\} \to [0,1]$, for $i = 1, ..., n$, that map voter i's signal s_i to a probability $p_i = \sigma_i(s_i)$ for i voting on alternative 1 (and voting on alternative 0 with the complementary probability, $1 - \sigma_i(s_i)$). Sincere voting thus is the strategy $\sigma_i(s_i) \equiv s_i$.

Consider now a given voter i who has received signal $s_i = 0$. Denote by $\mathcal{T}_i$ the event of a tie among all other votes. Such a tie may arise by chance, even for given signals, if other voters randomize their votes. However, since signals, and hence also votes, are statistically independent conditionally upon the state ω, we have, under any strategy profile $(\sigma_1, ..., \sigma_n)$:

$$\Pr[\mathcal{T}_i \ \wedge \ s_i = 0 \mid \omega] = \Pr[\mathcal{T}_i \mid \omega] \cdot \Pr[s_i = 0 \mid \omega]$$

for $\omega = 0, 1$. In the base-line model, game $G(m)$, the difference in expected utility for voter i between voting 0 and 1, conditional on having received signal 0, is

$$\begin{aligned}
\Delta u_i^0 &= \beta_i \Pr[\mathcal{T}_i \ \wedge \ \omega = 0 \mid s_i = 0] - \alpha_i \Pr[\mathcal{T}_i \ \wedge \ \omega = 1 \mid s_i = 0] \\
&= \beta_i \frac{(1-\mu_i)\, q_0}{\Pr[s_i = 0]} \Pr[\mathcal{T}_i \mid \omega = 0] - \alpha_i \frac{\mu_i(1-q_1)}{\Pr[s_i = 0]} \Pr[\mathcal{T}_i \mid \omega = 1]
\end{aligned}$$

In the game $G_\delta(m)$ perturbed with the sincerity bonus δ, voting sincerely when receiving signal 0 is optimal if and only if $\Delta u_i^0 + \delta \geq 0$, or, equivalently,

$$\delta \Pr[s_i = 0] + \beta_i (1-\mu_i) q_0 \Pr[\mathcal{T}_i \mid \omega = 0] - \alpha_i \mu_i (1-q_1) \Pr[\mathcal{T}_i \mid \omega = 1] \geq 0, \quad (26)$$

and mixing is optimal if and only if this last equation is an equality.

Likewise, the difference in expected utility for voter i between voting 1 and 0, conditional on having received signal 1, is

$$\Delta u_i^1 = \alpha_i \frac{\mu_i q_1}{\Pr[s_i = 1]} \Pr[\mathcal{T}_i \mid \omega = 1] - \beta_i \frac{(1-\mu_i)(1-q_0)}{\Pr[s_i = 1]} \Pr[\mathcal{T}_i \mid \omega = 0],$$

and thus sincere voting in this case is optimal if only if

$$\delta \Pr[s_i = 1] + \alpha_i \mu_i q_1 \Pr[\mathcal{T}_i \mid \omega = 1] - \beta_i (1-\mu_i)(1-q_0) \Pr[\mathcal{T}_i \mid \omega = 0] \geq 0, \quad (27)$$

and mixing is optimal if and only if this last equation is an equality.

Summing the right hand sides of (26) and (27) yields:

$$\delta + \beta_i (1-\mu_i)(2q_0 - 1) \Pr[\mathcal{T}_i \mid \omega = 0] + \alpha_i \mu_i (2q_1 - 1) \Pr[\mathcal{T}_i \mid \omega = 1],$$

Because q_0 and q_1 are larger than $1/2$, this is a strictly positive number as soon as δ is strictly positive. Therefore at least one of the two inequalities (26) and (27) is strict, which means that each voter must be voting sincerely on (at least) one signal. In particular no voter can be strictly mixing on both signals. The same argument works in games $\Phi_\varepsilon(m)$ and it is worth noticing this fact:

Fact: Each voter is voting sincerely on at least one signal.

From this it follows that there exists one signal , say signal $s = 0$, such that at least half of the population vote sincerely when receiving this signal. Without loss of generality we may take the point of view of individual $i = 2m+1$ and suppose that individuals $j = 1, ..., m$ vote $v_j = 0$ when receiving signal $s_j = 0$. Let $\mathcal{N}_0$ denote the random variable "number of votes 0 among voters $1, ..., 2m$, conditionally on $\omega = 0$". Then:

$$\Pr[\mathcal{T}_i \mid \omega = 0] = \Pr[\mathcal{N}_0 = m].$$

One can decompose the variable $\mathcal{N}_0$ as:

$$\begin{aligned}
\mathcal{N}_0 &= \mathcal{X}_0 + \mathcal{Y}_0 \\
\mathcal{X}_0 &= \sum_{j=1}^{m} \mathbf{1}\{s_j = 0 \mid \omega = 0\} \\
\mathcal{Y}_0 &= \sum_{j=1}^{m} \mathbf{1}\{s_j = 1 \wedge v_j = 0 \mid \omega = 0\} + \sum_{j=m+1}^{2m} \mathbf{1}\{v_j = 0 \mid \omega = 0\}
\end{aligned}$$

Notice that

$$\begin{aligned}\Pr[\mathcal{N}_0 = m] &= \sum_{k=0}^{m} \Pr[\mathcal{X}_0 = k] \cdot \Pr[\mathcal{Y}_0 = m-k] \\ &\leq \max_{0\leq k\leq m} \Pr[\mathcal{X}_0 = k]\end{aligned}$$

We do not know the probability distribution of $\mathcal{Y}_0$, because of possible mixing, but we

know that $\mathcal{X}_0$ is binomial with parameter q_0 and m. Therefore $\max_{0\leq k\leq m} \Pr[\mathcal{X}_0 = k]$ is equal to $\Pr[\mathcal{X}_0 = \lfloor q_0 m \rfloor]$ where $\lfloor q_0 m \rfloor$ denotes the integer part of $q_0 m$. If $q_0 m$ is an integer, then we obtain:

$$\Pr[\mathcal{N}_0 = m] \leq \Pr[\mathcal{X}_0 = q_0 m] = \binom{q_0 m}{m} q_0^{q_0 m}(1-q_0)^{m-q_0 m},$$

and, after using Stirling's formula:

$$\Pr[\mathcal{X}_0 = q_0 m] \sim \frac{1}{\sqrt{2\pi m q_0(1-q_0)}}.$$

This last property can be shown to actually hold even if $q_0 m$ is not an integer but we leave this technical point to the interested reader. To have a majorization, we may note for instance that it follows that there exists an A (which only depends on q_0) such that for all $m > A$, $\Pr\left[\mathcal{T}_{2m+1} \mid \omega = 0\right] < B/\sqrt{m}$, for $B = 1/\sqrt{q_0(1-q_0)}$. Thus, looking again at the condition (27) one can see that, for $m > A$ and

$$\delta > \frac{1}{\Pr[s=1]}\beta_i(1-\mu_i)(1-q_0)\frac{B}{\sqrt{m}} = \frac{\beta_i(1-\mu_i)(1-q_0)}{(1-q_1)\mu_i + (1-\mu_i)(1-q_0)}\frac{B}{\sqrt{m}}$$

(27) is a strict inequality, which means that sincere voting on signal $s = 1$ is strictly optimal for the considered voter $i = 2m+1$. Then it follows that there exists a B' (which depends on all the parameters q, α, β, μ) such that for $m > A$ and $\delta > B'/\sqrt{m}$, sincere voting on signal $s = 1$ is strictly optimal for all voters. The values of the parameters α_i, β_i, μ_i for different voters are bounded, so we can take B' to be a constant of the model, independent of the population size.

If all voters vote sincerely on signal $s = 1$, the number $\mathcal{N}_1$ of votes 1 among voters

$j = 1, ...2m$, conditionally on $\omega = 1$ can be decomposed as:

$$\begin{aligned}
\mathcal{N}_1 &= \mathcal{X}_1 + \mathcal{Y}_1 \text{ for} \\
\mathcal{X}_1 &= \sum_{j=1}^{2m} \mathbf{1}\{s_j = 1 \mid \omega = 1\} \\
\mathcal{Y}_1 &= \sum_{j=1}^{2m} \mathbf{1}\{s_j = 0 \wedge v_j = 1 \mid \omega = 1\}
\end{aligned}$$

with $\mathcal{X}_1$ binomial $(2m, q_1)$. Again we note that

$$\begin{aligned}
\Pr\left[\mathcal{T}_i \mid \omega = 1\right] &= \Pr[\mathcal{N}_1 = m] = \Pr[\mathcal{X}_1 + \mathcal{Y}_1 = m] \\
&= \sum_{k=0}^{m} \Pr[\mathcal{X}_1 = k] \cdot \Pr[\mathcal{Y}_1 = m - k] \\
&\leq \max_{0 \leq k \leq m} \Pr[\mathcal{X}_1 = k]
\end{aligned}$$

The mode of the binomial distribution of $\mathcal{X}_1$ is reached at the integer part of $2q_1 m$, a number that exceeds m. It follows that

$$\max_{0 \leq k \leq m} \Pr[\mathcal{X}_1 = k] = \Pr[\mathcal{X}_1 = m] = \binom{m}{2m} q_0^m (1 - q_0)^m.$$

Using Stirling's approximation formula, one finds again that this number is decreasing (this time exponentially) with m. The same reasoning as before can now take place with respect to equation (26): the negative term, $-\alpha_i \mu_i (1 - q_1) \Pr\left[\mathcal{T}_i \mid \omega = 1\right]$, is asymptotically small and we conclude that there exist numbers A' and B'' such that if $m > A'$ and $\delta > B'/\sqrt{m}$, inequalities (26) and (27) are both strict for all i, which means that all voters vote sincerely on both signals. Point (ii) in Theorem 4 follows immediately. The reasoning is the same for Theorem 5.

9.10. Claim (ii) in Theorems 4 and 5, symmetric mixed equilibria. We proved above that the lower bound $\bar{\delta}_n$ can be taken to zero exponentially with n, instead of at the rate $1/\sqrt{n}$, when we restrict the uniqueness claim to pure-strategy equilibria. We here prove that this also holds for symmetric mixed equilibria.

Consider first a game $G_\delta(m)$ and suppose that voter i votes sincerely when receiving the signal $s_i = 0$ (that is, $\sigma_i(0) = 0$) but is mixing on signal $s_i = 1$ (that is, $0 < \sigma_i(1) < 1$). Then one has $\Delta(u_i \mid s_i = 0) \geq 0$ and $\Delta(u_i \mid s_i = 1) = 0$. Hence, with the sincerity bonus δ:

$$\begin{aligned}
\alpha_i \mu_i (1 - q_1) \Pr\left[\mathcal{T} \mid \omega = 1\right] - \beta_i \left(1 - \mu_i\right) q_0 \Pr\left[\mathcal{T} \mid \omega = 0\right] &\leq \delta \Pr\left[s_i = 0\right] \\
\alpha_i \mu_i q_1 \Pr\left[\mathcal{T} \mid \omega = 1\right] - \beta_i \left(1 - \mu_i\right)\left(1 - q_0\right) \Pr\left[\mathcal{T} \mid \omega = 0\right] &= -\delta \Pr\left[s_i = 1\right]
\end{aligned}$$

We now focus on symmetric equilibria: $\sigma_j(0) = 0$ and $\sigma_j(1) = \sigma_1 < 1$ for all voters j. It is easily shown that then

$$\begin{aligned}
\Pr[\mathcal{T} \mid \omega = 1] &= \binom{m}{2m} A^m \\
\Pr[\mathcal{T} \mid \omega = 0] &= \binom{m}{2m} B^m
\end{aligned}$$

with

$$\begin{aligned}
A &= q_1\sigma_1(q_1(1-\sigma_1) + 1 - q_1) = q_1\sigma_1(1 - q_1\sigma_1) \\
B &= (1-q_0)\sigma_1(q_0 + (1-q_0)(1-\sigma_1)) = (1-q_0)\sigma_1(1-(1-q_0)\sigma_1)
\end{aligned}$$

so that the indifference equation can be written:

$$F(\sigma_1) = \phi^*$$

where the function $F : [0,1] \to \mathbb{R}$ is defined by

$$F(\sigma_1) = -\alpha_i\mu_i q_1 A^m + \beta_i(1-\mu_i)(1-q_0)B^m$$

and

$$\phi^* = \delta \binom{m}{2m}^{-1} \Pr[s_i = 1]$$

Let us study this function F. We only consider values of m and δ such that sincere voting is a strict equilibrium (and we know that this is possible for exponentially small δ). Then at $\sigma_1 = 1$, we know that $F(1) < \phi^*$ because if the others are not mixing, then it is a strict best response for i to vote according to her signal $s_i = 1$. It is easy to see that $F(0) = 0$.

We claim that there is no solution to the equation $F(\sigma_1) = \phi^*$. The function F is polynomial, with $F(0) < \phi^*$ and $F(1) < \phi^*$. Hence, if $F(\sigma_1) = \phi^*$ has an interior, then the maximum of F must be attained at some interior point σ_1^*, at which $F'(\sigma_1^*) = 0$ and $F(\sigma_1^*) \geq \phi^* > 0$. However,

$$F'(\sigma_1) = -\alpha_i\mu_i q_1 m A^m \frac{A^*}{A} + \beta_i(1-\mu_i)(1-q_0)mB^m \frac{B^*}{B}$$

with

$$\begin{aligned}
\frac{\sigma_1 A^*}{A} &= \frac{1 - 2q_1\sigma_1}{1 - q_1\sigma_1} \\
\frac{\sigma_1 B^*}{B} &= \frac{1 - 2(1-q_0)\sigma_1}{1 - (1-q_0)\sigma_1}
\end{aligned}$$

For all σ_1, $B^* > 0$. For $\sigma_1 \geq \frac{1}{2q_1}$, $A^* \leq 0$ and thus $F'(\sigma_1) > 0$. For $\sigma_1 < \frac{1}{2q_1}$, $A^* \geq 0$. Note that $1 - q_0 < 1/2 < q_1$ implies

$$\frac{A^*}{A} < \frac{B^*}{B}.$$

Therefore:

$$\begin{aligned} F'(\sigma_1) &= -\alpha_i \mu_i q_1 m A^m \frac{A^*}{A} + \beta_i (1 - \mu_i)(1 - q_0) m B^m \frac{B^*}{B} \\ &> m \frac{A^*}{A} (-\alpha_i \mu_i q_1 A^m + \beta_i (1 - \mu_i)(1 - q_0) B^m) \\ &= m \frac{A^*}{A} F(\sigma_1) \end{aligned}$$

and thus in this case we would have $F'(\sigma_1^*) > m \frac{A^*}{A} F(\sigma_1^*) > \phi^* > 0$, a contradiction.

The same argument works for games $\Phi_\varepsilon(m)$, establishing that, for exponentially small perturbations there is no symmetric mixed-strategy equilibrium.

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